# Tunable excitons in gated graphene systems

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## ABSTRACT

We develop a microscopic theory of a strong electromagnetic radiation interaction with bilayer graphene where an energy gap is opened by a static electric field perpendicular to graphene planes. We show that an adiabatic variation of the gate potentials (that leads to the resonance of the energy gap with electromagnetic field) may produce full inversion of the electron population between valence and conduction bands. Quantum kinetic equations for density matrix are obtained by using a tight-binding approach within second quantized Hamiltonian in an intense laser field and by taking into account Coulomb correlations between particles. Excitonic absorption in graphene systems (monolayer and bilayer) with opened energy gap is also investigated for different values of the gap and the parameters describing the band structure.

**Keywords:** monolayer and bilayer graphene, intensive electromagnetic radiation, population inversion, bound excitonic states

## 1. Introduction

Graphene is a single layer of graphite with honeycomb lattice structure. Since its experimental discovery and isolation [1], over the past decade, graphene has attracted enormous interest due to its extraordinary electronic properties [1, 2].
Two-dimensional crystals exhibit fascinating electrical, mechanical, optical properties and are very promising e.g. for nanoelectronics [3] and as transparent conducting layers.
Massless Dirac fermions are present in graphene; the role of the speed of light $c$, is here played by the parameter $v_F \approx 10^6 m/s$, which is smaller than $c$ by two orders of magnitude.
Graphene is a unique bridge between condensed matter physics and relativistic quantum field theory and due to its unique properties is of great interest for nonlinear optical applications [4, 5]. The nonlinear quantum electrodynamics effects can be observed in graphene already for fields available in the laboratory [6].
The interest in the optical response of graphene is boosted by recent progress of terahertz radiation technology, which is another frontier research area [4].
Theoretical and experimental investigations on the nonlinear effects induced in a graphene system have so far been mainly focused in monolayer graphene. Meanwhile, in the physics of graphene there is growing interest in bilayer and trilayer graphene systems, where the electronic band structures are richer than in monolayer graphene [7, 8] and can be easily manipulated by external field.
Theoretical and experimental investigations have shown that a perpendicular electric field applied to bilayer of graphene modifies its band structure near the K point and may open an energy gap in the electronic spectrum, which is tunable by the gate voltage [9]. Magnetotransport experiments [10] showed that the induced gap between the conduction and valence bands could be tuned between zero and midinfrared energies. Moreover, the magnitude of the gap was found to strongly depend on the number of graphene layers and its stacking order. In [11, 12] we studied theoretically electric field induced band gap of graphene multilayers with different ways of stacking between consecutive graphene planes.
So, a desirable band structure for multilayer graphene systems, that can be useful for different goals of nano- and optoelectronics, can be theoretically found (and suggested for an experimental realization) e.g. by corresponding choice of the layers number in a graphene system and by application of an external field of corresponding magnitude.
In this work, on the basis of a tight-binding approach, we investigate the nonlinear optical response of monolayer and bilayer graphene systems to an intense laser field. In addition to fundamental interest, intrinsic to such study, the

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understanding of the nonlinear optical properties of graphene systems may open the way for new applications in nano- and optoelectronics.

We develop a microscopic theory of a strong electromagnetic radiation interaction with monolayer and bilayer graphene systems with opened energy gap. We consider one-photon resonant interaction of a laser field with bilayer graphene, with coherent superposition states induced by laser radiation. Our analysis is based on a tight-binding approach and second quantized Hamiltonian in an intense laser field.

In order to understand the influence of Coulomb interaction and impact of many body physics in graphene systems it is interesting to study excitonic absorption in monolayer and bilayer graphene systems with opened energy gap. In gapless monolayer graphene, the Coulomb problem has no true bound states, but resonances [13]. The optical response of graphene with an opened energy gap between the conduction and valence bands is dominated by bound excitons [14, 15]. Furthermore, since Coulomb interactions can be controlled externally by choosing the dielectric constant of the substrate on which graphene is deposited, it is possible to tune the electron-electron interactions in a way that cannot be done in most materials.

In this work we also study excitonic absorption in bilayer graphene with energy gap (opened by static electric field) in the field of laser radiation. For Coulomb problem, we consider only low excitation regime, where we only have a small density of electrons and holes in the system. The excitonic absorption in a gapped graphene monolayer in linear approach is also studied. A substantial band gap in monolayer graphene (about of 0.2 eV) can be induced in several ways, e.g., by coupling to substrates [16], electrical biasing, or nanostructuring [2, 16].

The plan of the paper is as follows. Section 2 presents the theory of the laser interaction with a gated graphene system in an external electric field. In Section 2.1 the evolution equations for the single-particle density matrix are obtained. Coulomb correlations and excitonic absorption in monolayer and bilayer graphene systems are investigated in Section *2.2.* Section 3 contains our numerical results together with a discussion of essential physical points. Section 4 presents our main conclusions.

## 2. Theory

We consider the interaction of a strong electromagnetic wave with bilayer graphene, first ignoring electron-electron interactions. A perpendicular electric field created by top and back gates [11, 12] opens an energy gap in multilayer graphene. We assume that the laser pulse propagates in the perpendicular direction to graphene plane (XY) and the electric field ***E*** (t) of pulse lies in the graphene plane.

The Hamiltonian in the second quantization formalism, in the absence of e-e interactions, has the form

$$H = \int \Psi^{+} H_s \Psi d\mathbf{r}, \tag{1}$$

The Hamiltonian $H_s$ for bilayer in the vicinity of the *K* point can be written as (here we omit the real spin and valley quantum numbers):

$$H_s = H_0 + H_d \tag{2}$$

The first term in Eq. (2) corresponds to bilayer graphene in the field of perpendicular electric field, and the second term is the interaction Hamiltonian between a laser field and bilayer graphene. The forms of $H_0$ and $H_d$ are given further below, where $v_3 \approx 0.1 v_F$ is the effective velocity ($v_F$ is the Fermi velocity in monolayer graphene), *U* is the gap introduced by the perpendicular electrical field, and $\mathbf{p} = \{p_x, p_y\}$ is the electronic momentum:

$$H_0 = \begin{pmatrix} -U/2 & v_3(p_x + ip_y) - (p_x - ip_y)^2/2m \\ v_3(p_x - ip_y) - (p_x + ip_y)^2/2m & U/2 \end{pmatrix}$$

$$H_d = \begin{pmatrix} e\mathbf{r}\cdot\mathbf{E}(t) & 0 \\ 0 & e\mathbf{r}\cdot\mathbf{E}(t) \end{pmatrix}.$$

We expand the fermionic field operator $\Psi(\mathbf{r},\mathrm{t})$ over the free wave function $\psi_\sigma(\mathbf{p})$ of bilayer graphene

$$\Psi(\mathbf{r},\mathrm{t}) = \sum_{\mathbf{p},\sigma} \hat{a}_{\mathbf{p},\sigma}(t)\psi_\sigma(\mathbf{p})\, \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\mathbf{pr}} / \sqrt{S}\,, \tag{3}$$

where the annihilation operator $\hat{a}_{\mathbf{p},\sigma}(t)$ is associated with positive and negative energy solutions $\sigma = \pm 1$, and $S$ is the area of graphene layer. Introducing $\theta(\mathbf{p}) = \arctan(p_y / p_x)$, $p_x + ip_y = p\exp(i\theta)$ the expression for the energy spectrum of bilayer graphene can be presented in the form

$$\varepsilon_{\mathbf{p},\sigma} = \sigma\sqrt{\frac{U^2}{4} + (\mathrm{v}_3 p)^2 - \frac{\mathrm{v}_3 p^3}{m}\cos 3\theta + (\frac{p^2}{2m})^2}\,. \tag{4}$$

The free solutions in bilayer graphene $\psi_\sigma(\mathbf{p})$ have the following form:

$$\psi_\sigma(\mathbf{p}) = \frac{\sqrt{\varepsilon_{\mathbf{p}\sigma} + U/2}}{\sqrt{2\varepsilon_{\mathbf{p}\sigma}}}\begin{pmatrix} 1 \\ \dfrac{1}{\varepsilon_{\mathbf{p}\sigma} + U/2}\gamma(p,\theta) \end{pmatrix}, \tag{5}$$

with $\gamma(p,\theta) = -\dfrac{p^2}{2m}\exp(i2\theta) + v_3 p\exp(-i\theta)$.

## 2.1 Laser interaction with bilayer graphene system in an external electric field

Taking into account Eqs. (1-3) the second quantized Hamiltonian for the single-particle part can be expressed in the form:

$$\hat{\mathrm{H}} = \sum_{\mathbf{p},\sigma} \varepsilon_\sigma(p)\hat{a}^+_{\mathbf{p},\sigma}\hat{a}_{\mathbf{p},\sigma} + e\mathbf{E}(t)\sum_{\mathbf{p},\sigma}\sum_{\mathbf{p}',\sigma'} \mathbf{D}_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')\hat{a}^+_{\mathbf{p},\sigma}\hat{a}_{\mathbf{p}',\sigma'}\,. \tag{6}$$

For the dipole matrix element

$$\mathbf{d}_{\sigma\sigma'}(\mathbf{p},\mathbf{p}') = i\,e\,\mathbf{D}_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')\,, \tag{7}$$

where

$$\mathbf{D}_{\sigma\sigma'}(\mathbf{p},\mathbf{p}') = \psi^+{}_\sigma(\mathbf{p})\,\psi_\sigma(\mathbf{p}')\int \mathbf{r}\, e^{\frac{i}{\hbar}(\mathbf{p}'-\mathbf{p})\mathbf{r}}\, d\,\mathbf{r} / S\,, \tag{8}$$

we obtained an expression, which we do not bring here due to its long form. In contrast to the non-relativistic case, we found that the dipole matrix element for the light interaction with the bilayer depends on electron momentum.
We then use Heisenberg picture, where operator evolution is given by the following equation

$$i\,\hbar\frac{\partial \hat{L}}{\partial t} = [\hat{L},\hat{H}]\,, \tag{9}$$

and this is applied to the single-particle density matrix in momentum space, that is defined as:

$$\rho_{\sigma\sigma'}(\mathbf{p},\mathbf{p}') = <\hat{a}^+_{\mathbf{p},\sigma}\ \hat{a}_{\mathbf{p}',\sigma'}>\,. \tag{10}$$

Using Eqs. (6-10) one can arrive to the evolution equation for the single-particle density matrix for different $\mathbf{p}$ values, and in the field of external laser field:

$$i\hbar\,\partial\rho_{1-1}(\mathbf{p},t)/\partial t = 2\varepsilon_{\mathbf{p}1}\,\rho_{1-1}(\mathbf{p},t) + \mathbf{E}(t)\mathbf{d}(\mathbf{p})[2\rho_{11}(\mathbf{p},t)-1], \tag{11}$$
$$i\hbar\,\partial\rho_{11}(\mathbf{p},t)/\partial t = [\rho_{1\,-1}(\mathbf{p},t)\mathbf{E}^{*}(t)\mathbf{d}^{*}(\mathbf{p}) - \rho_{-1\,1}(\mathbf{p},t)\mathbf{E}(t)\mathbf{d}(\mathbf{p})],$$

where the index $\sigma = 1$ is connected with the conduction band and the index $\sigma = -1$ is related to the valence band.

## 2.2. Coulomb correlations and excitonic absorption in gapped graphene systems

The Hamiltonian for bilayer graphene in the second quantization formalism in the presence of electron-electron interaction has the form

$$H = \int \Psi^{+} H_s \Psi d\mathbf{r} + H_{Coul} \tag{12}$$

$$H_{Coul} = \frac{1}{2}\int\int \Psi^{+}(\mathbf{r})\Psi^{+}(\mathbf{r}')\,V(\mathbf{r}-\mathbf{r}')\,\Psi\ (\mathbf{r}')\Psi\ (\mathbf{r})d\mathbf{r}d\mathbf{r}'. \tag{13}$$

The Hamiltonian $H_s$ in Eq. (12) is given by Eq. (2) and $H_{Coul}$ is two-particle Coulomb Hamiltonian. Using the expansion of the fermionic field operator over the annihilation and creation operators Eq.(3), we obtain the expression for total Coulomb Hamiltonian, that consists of the four terms

$$H_{Coul} = H_I + H_{II} + H_{III} + H_{IV}$$

We omit valley quantum numbers, and suppress the spin index $s$ from now on. The following calculations do not depend on $s$, only the spin summation leads to an extra 2 factor in the final result for macroscopic interband polarization that is induced by the coherent monochromatic light field. Using macroscopic interband polarization we compute the optical susceptibility from which we then get the absorption coefficient and the refractive index.

Due to its complexity, we bring here the expression only for the first term of $H_{Coul}$

$$\begin{aligned} H_{Coul\,I} = \frac{1}{2}\sum_{\mathbf{k}'\mathbf{k}''\mathbf{q}} V_{2D}(q)\{&\hat{c}^{+}_{\mathbf{k}'-\mathbf{q}}\hat{c}^{+}_{\mathbf{k}''+\mathbf{q}}\hat{c}_{\mathbf{k}''}\hat{c}_{\mathbf{k}'}f^{+}(\mathbf{k}'-\mathbf{q})f^{+}(\mathbf{k}''+\mathbf{q})f(\mathbf{k}'')f(\mathbf{k}') \\ &+\hat{c}^{+}_{\mathbf{k}'-\mathbf{q}}\hat{c}^{+}_{\mathbf{k}''+\mathbf{q}}\hat{c}_{\mathbf{k}''}\hat{v}_{\mathbf{k}'}f^{+}(\mathbf{k}'-\mathbf{q})f^{+}(\mathbf{k}''+\mathbf{q})f(\mathbf{k}'')f_v(\mathbf{k}') \\ &+\hat{c}^{+}_{\mathbf{k}'-\mathbf{q}}\hat{c}^{+}_{\mathbf{k}''+\mathbf{q}}\hat{v}_{\mathbf{k}''}\hat{c}_{\mathbf{k}'}f^{+}(\mathbf{k}'-\mathbf{q})f^{+}(\mathbf{k}''+\mathbf{q})f_v(\mathbf{k}'')f(\mathbf{k}') \\ &+\hat{c}^{+}_{\mathbf{k}'-\mathbf{q}}\hat{c}^{+}_{\mathbf{k}''+\mathbf{q}}\hat{v}_{\mathbf{k}''}\hat{v}_{\mathbf{k}'}f^{+}(\mathbf{k}'-\mathbf{q})f^{+}(\mathbf{k}''+\mathbf{q})f_v(\mathbf{k}'')f_v(\mathbf{k}')\} \end{aligned} \tag{14}$$

Here we define the operator $\hat{a}^{+}_{\mathbf{k},\sigma}$ in the conduction band as $\hat{c}^{+}_{\mathbf{k}}(t)$, and the operator $\hat{a}_{\mathbf{k},\sigma}$ in the valence band as $\hat{v}_{\mathbf{k}}(t)$ and $f(\mathbf{k})$ is the value of the wave function Eq.(5) in the specific $\mathbf{k}$ point.

In order to obtain the Coulomb Hamiltonian for gapped graphene monolayer, we use the same procedure described above for graphene bilayer. Now, in single particle Hamiltonian $H_s$ (see Eq. (2)) in the vicinity of the *K* point, $H_0$ has the form

$$H_0 = \begin{pmatrix} -U/2 & \gamma_0(k_x - i\,k_y) \\ \gamma_0(k_x + i\,k_y) & U/2 \end{pmatrix}. \tag{15}$$

The potential energy difference, that opens an energy gap $U$, can be achieved by the inversion symmetry breaking. Experimentally, it can be obtained by placing graphene onto a substrate in which the A and B atoms experience different on-site energies. Monolayer graphene grown epitaxially on SiC substrate has a band gap of about 0.2 eV [16].

The expression for the energy spectrum of monolayer graphene is:

$$\varepsilon_{k\sigma} = \sigma\sqrt{U^2/4 + (\gamma_0 k)^2} \tag{16}$$

The free solutions in gapped monolayer graphene $\psi_\sigma(\mathbf{p})$ have the following form

$$\psi_\sigma(\mathbf{p}) = \frac{\sqrt{\varepsilon_{k\sigma} + U/2}}{\sqrt{2\varepsilon_{k\sigma}}}\begin{pmatrix} 1 \\ v_F \hbar k \exp(i\theta)/(\varepsilon_{k\sigma} + U/2) \end{pmatrix}, \tag{17}$$

$$\psi^+{}_\sigma(\mathbf{p}) = \frac{\sqrt{\varepsilon_{k\sigma} + U/2}}{\sqrt{2\varepsilon_{k\sigma}}}\left(1, \quad v_F \hbar k \exp(-i\theta)/(\varepsilon_{k\sigma} + U/2)\right),$$

where $\theta(p) = \arctan(p_y/p_x),\ p_x + ip_y = p\exp(i\theta)$.

In the case of graphene monolayer, $H_{Coul}$ has the same form (see Eq.(13)), the difference being now $f(\mathbf{k})$ is the value of the wave function Eq.(17 ) in the specific $\mathbf{k}$ point.

As we see, the first term of Coulomb Hamiltonian $H_{Coul\,I}$, consists of four terms: so the total $H_{Coul}$ contains 16 terms. The second term of Coulomb Hamiltonian $H_{Coul\,II}$ can be obtained from Eq. (14) by changing in the second column the operator related to the conduction band $\hat{c}^+_{\mathbf{k''}+\mathbf{q}}$ by the valence one $\hat{v}^+_{\mathbf{k''}+\mathbf{q}}$. These operators satisfy the anticommutation rules

$$[\hat{c}^+_{\mathbf{k}}\hat{c}_{\mathbf{k'}}]_+ = \delta_{k,k'},\ [\hat{c}_{\mathbf{k}}\hat{c}_{\mathbf{k'}}]_+ = 0,\ [\hat{c}^+_{\mathbf{k}}\hat{v}_{\mathbf{k'}}]_+ = 0\ .$$

To take into account the contribution of the Coulomb interaction in Eq. (9) for an operator evolution, we use now the total Hamiltonian given by Eq. (13).

The product of four field operators describes all many particle correlations as trions, biexcitons, etc. To take into account only excitonic effects, we apply the Hartree-Fock approximation to the many particle system, i.e. we express four field operator averages in Coulomb Hamiltonian as products of the polarization and population, e.g. for $< \hat{v}^+_{\mathbf{k}}\ \hat{c}_{\mathbf{k'}+\mathbf{q}}\hat{c}_{\mathbf{k}+\mathbf{q}}\hat{c}_{\mathbf{k'}} >$

$$< \hat{v}^+_{\mathbf{k}}\ \hat{c}_{\mathbf{k'}+\mathbf{q}}\hat{c}_{\mathbf{k}+\mathbf{q}}\hat{c}_{\mathbf{k'}} > = < \hat{v}^+_{\mathbf{k}}\ \hat{c}_{\mathbf{k'}} >< \hat{c}_{\mathbf{k'}+\mathbf{q}}\hat{c}_{\mathbf{k}+\mathbf{q}} > \delta_{\mathbf{k},\mathbf{k'}} - < \hat{v}^+_{\mathbf{k}}\ \hat{c}_{\mathbf{k}+\mathbf{q}} >< \hat{c}_{\mathbf{k'}+\mathbf{q}}\hat{c}_{\mathbf{k'}} > \delta_{\mathbf{k},\mathbf{k}+\mathbf{q}} \tag{18}$$

Using this approximation, we truncate the infinite Bogolubov chain, and obtain closed set of equations which leads to exact exciton energy. Using Eqs. (6)- (14) one can arrive to the evolution equation for the single-particle density matrix for different values of $k_x, k_y$ and in the field of external laser field.

$$i\hbar\,\partial\rho_{1-1}(\mathbf{k},t)/\partial t = \Sigma(\mathbf{k})\rho_{1-1}(\mathbf{k},t) + \Lambda(\mathbf{k})[2\rho_{11}(\mathbf{k},t) - 1] \tag{19}$$

$$i\hbar\,\partial\rho_{11}(\mathbf{k},t)/\partial t = [\rho_{1-1}(\mathbf{k},t)\Lambda^*(\mathbf{k}) - \rho_{-11}(\mathbf{k},t)\Lambda(\mathbf{k})]$$

where for renormalized energy $\Sigma(\mathbf{k})$ and Rabi frequency $\Lambda(\mathbf{k})/\hbar$ we obtain the relations

$$\Sigma(\mathbf{k}) = 2\varepsilon_{\mathbf{k}1} + \sum_{\mathbf{q}\neq 0} V_{2D}(q)\,T(\varepsilon_k, U, \rho_{1-1}(\mathbf{k}), \rho_{11}(\mathbf{k})) \tag{20}$$

$$\Lambda(\mathbf{k}) = \mathbf{E}(t)\mathbf{d}(\mathbf{k}) + \sum_{\mathbf{q}\neq 0} V_{2D}(q)\mathrm{P}(\varepsilon_k, U, \rho_{1-1}(\mathbf{k}), \rho_{11}(\mathbf{k}))$$

We do not bring the obtained analytical expressions for T and $P$ functions (in monolayer as well as in bilayer case) because of their complicated forms. Here $V_{2D}(q)$ is the Fourier transform of two-dimensional Coulomb potential, and the electric field of laser pulse has the following form $E(t) = \exp(-i\omega t)\exp[-(t/\tau)^2]$, with $\tau >> 2\pi/\omega = T$. We solve obtained integro-differential equations numerically using Runge–Kutta method with time step $\Delta t < T$. As a result, we obtain the interband polarization as a function of time, and define the macroscopic polarization that includes contributions from different values of $\mathbf{k}$: $P(t) = \sum_{\mathbf{k}} P_{\mathbf{k}}(t) d^*(\mathbf{k}) + c.c.$

Using then Fourier transform of macroscopic interband polarization, we compute the optical susceptibility, and get the absorption coefficient as complex part of the optical susceptibility. In the numerical calculations we introduced dumping constant about 0.2 effective Rydberg.

## 3. Results and Discussion

In the present paper the microscopic theory of a strong electromagnetic radiation interaction with multilayer graphene systems is developed. We consider one-resonant interaction of a laser field with bilayer graphene when an energy gap $U$ is opened due to external gates. As we will see below, we find that changing the energy gap linearly on time, the electron population is transferred from the top of valence band to the bottom of conduction one after the time $t_f$, when the gap comes into resonance with the electromagnetic field. This is an alternative and more suitable way to bring the system into the resonance in comparison with the method of frequency chirped pulse. Figure 1 shows 2D plot for the electron distribution $\rho_{11} = N_c$ after the interaction with the laser pulse with $\hbar\omega_0 = 8E_L$ as a function of dimensionless momentum components (in units of the Lifshitz energy $E_L = mv_L^2/2 = 1\text{meV}$ and momentum $p_L$).

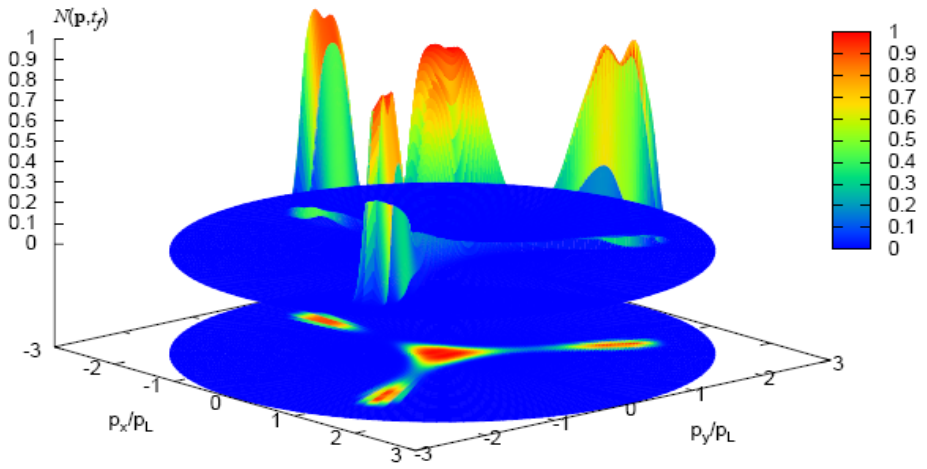


**Fig.1.** (Colour online) Particle distribution function $N_c(p)$ (in arbitrary units) after the interaction with the pulse having $\hbar\omega_0 = 8E_L$ (when $\hbar\omega_0 \approx U_{fin}$) as a function of scaled dimensionless moment components.

Figure 2 shows the electron distribution function $\rho_{11} = N_c$ in the conduction band for $\hbar\omega_0 = 50E_L$. We see that for larger value of the frequency the red triangle is larger, and correspondingly a larger number of electrons are transferred to the conduction band. It is connected with the fact that for larger value of the gap (see Fig. 2); the bilayer has more flat bands near the Fermi level. Estimations show that the density of electrons in the triangle is about $10^{11} cm^{-2}$. The high density of excitons can lead to Bose-Einstain condensation phenomenon.

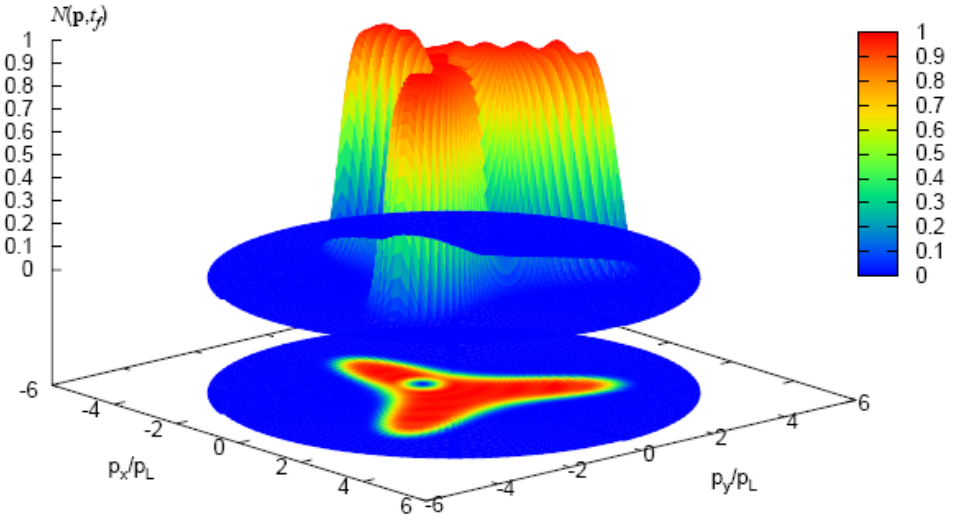


**Fig.2.** (Colour online) Particle distribution function $N_c(p)$ (in arbitrary units) after the interaction with the pulse for $\hbar\omega_0 = 50E_L$ ( when $\hbar\omega_0 \approx U_{fin}$ ) as a function of scaled dimensionless moment components.

The obtained 2D plot for the evolution of particle distribution function $\rho_{cc}(\mathbf{p},\mathbf{p}) = < \hat{a}^+_{\mathbf{p},\sigma} \hat{a}_{\mathbf{p},\sigma} > = N_c$ (in the conduction band) as a function of time is shown in Fig. 3 for the pulse having $\hbar\omega_0 = 32E_L$ with $\omega_0$ its frequency (in units of the Lifshitz energy $E_L = mv_L^2/2 = 1\text{meV}$ and momentum $p_L = mv_L$ ). For this case, during the interaction time $t_f$=100T, the energy gap of bilayer graphene reaches its maximal final value $U_{fin} = 28E_L$. For this case, i.e. for $\hbar\omega_0 > U_{fin}$, the electrons transfer to the conduction band in the region which is higher than the bottom of the conduction band into the isoenergetic line that has trigonal form (see the band structure for multilayers of graphene in Ref. (12) where we found that the bands has trigonal shape).
As shown in Fig.3, in the beginning of the interaction, the population of electrons in the conduction band is negligible (that corresponds to blue contour in Fig.3) while at the end of the interaction we observe the full inversion of the population of electrons between the valence and the conduction bands (that corresponds to the red contour in Fig.3).

Our numerical results for excitonic absorption in monolayer as well as in bilayer graphene systems with opened energy gap are obtained in the linear regime on the basis of Eqs (19, 20). Figure 4 shows excitonic absorption spectrum of gated monolayer graphene with the energy gap 500 meV as a function of detuning $\beta = (\hbar\omega - U)/\mathrm{R}^*$ where $R^* = \mu e^4 / 2\hbar^2\chi^2$ the effective Rydberg energy is. We introduce the reduced effective electron-hole mass by the expression $\mu = U/4v_F^2$ as proposed in Ref.15, so the mass is proportional to the band gap as in a simple two-band model .

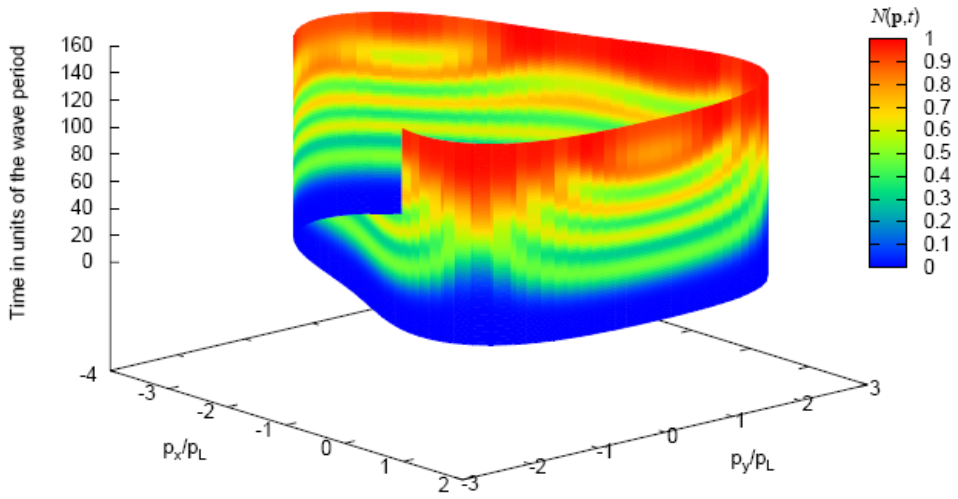


**Fig.3.** (Colour online) The evolution of the particle distribution function *Nc(p)* (in arbitrary units) during the interaction with the pulse with $\hbar\omega_0 = 32E_L$, and the energy gap of bilayer graphene reaches its maximal final value $U_{fin} = 28E_L$ (red color corresponds to the maximum of the population . i.e. *Nc(p)*=1).

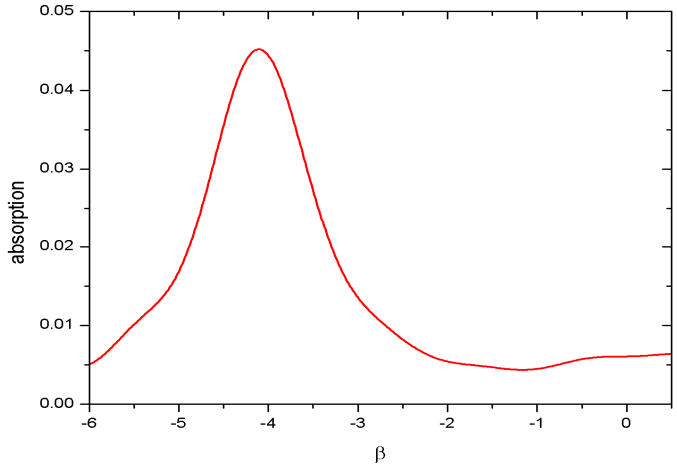


**Fig.4.** Excitonic absorption in monolayer graphene as a function of the detuning $\beta = (\hbar\omega - U)/R^*$ for the value of effective fine structure constant $\alpha = 0.175$.

In contrast with the standard semiconductor case, in graphene one deals with the fine structure constant $\alpha = \chi e^2/\hbar c$, and the appearance of bound states strongly depends on the value of $\alpha$ (the choice of dielectric constant $\kappa$ strongly affects this). For $\alpha$ =0.175 (for the value of dielectric constant $\chi = 12.5$) we obtained the maximum of the excitonic peak at $4.12R^*$ (see Fig.4). Notably, we obtained a good agreement with the exact analytical solution for relativistic 2D hydrogen atom [17]. Indeed, the expression for the ground state energy [17] has the form $E = m_e c^2\left[1+4\alpha^2/\left(1-4\alpha^2\right)\right]^{-1/2}$ which gives $E \approx 4.13R^*$ for the value of $\alpha = 0.175$. We found that the width of the peak in Fig. 4 is much larger than in the nonrelativistic case.

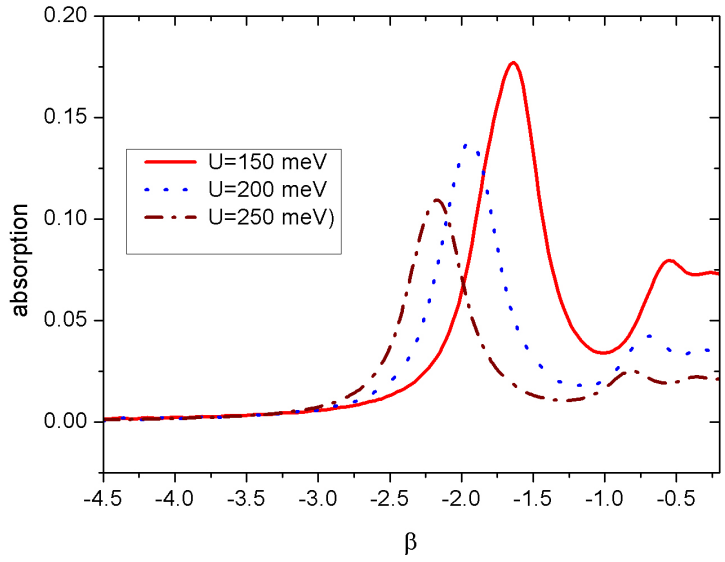


**Fig.5.** Excitonic absorption in bilayer graphene as a function of the detuning $\beta = (\hbar\omega - U)/R^*$ for different values of the gap: red (solid) line is for U=150 meV; blue (dotted) line corresponds to U=200meV. and brown (dash-dotted) line is for U=250meV.

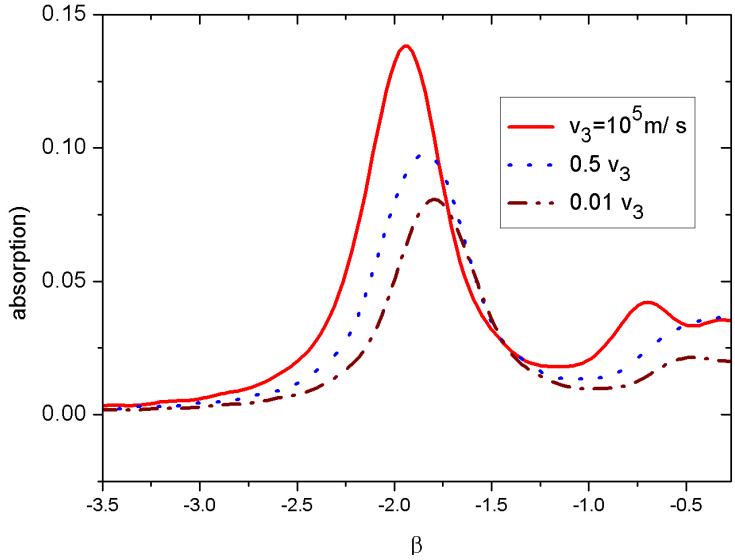


**Fig.6.** Excitonic absorption in bilayer graphene as a function of the detuning $\beta = (\hbar\omega - U)/R^*$ for different values of the parameter $v_3$ : red (solid) line is for $v_3 = 10^5 m/s$ ; blue (dotted) line corresponds to $v_3 = 0.5 \cdot 10^5 m/s$ and brown (dash-dotted) line is for $v_3 = 0.01 \cdot 10^5 m/s$ .

In Fig.5 we present our results obtained for excitonic absorption of gated graphene bilayer as a function of the detuning parameter $\beta = (\hbar\omega - U)/R^*$ for different values of opened energy gap $U$ . We found that the excitonic absorption

increases with decreasing of the gap value. This is connected with the fact that the coefficient in dipole matrix element obtained in this work is inversely proportional to the value of opened energy gap $U$. The position of the maximum of absorption curve corresponds to the exciton binding energy.

For bilayer graphene we introduce the effective electron mass in the energy spectrum Eq. (4) by the relation $m = \gamma_1 / 2\mathrm{v}_F^2$ [2, 9] with $\gamma_1 = 377\ meV$ being the vertical interlayer hopping parameter.

Our numerical results obtained for the excitonic absorption of gated graphene bilayer as a function of detuning $\beta = \left(\hbar\omega - U\right)/R^*$ for different values of effective velocity $\mathrm{v}_3 = \sqrt{3}\gamma_3 a / 2\hbar$ ($\gamma_3$ describes the interaction between $B$ atoms in the neighboring layers, and $a$ is the lattice constant) are presented in Fig.6.

The red solid curve in figure 6 corresponds to the value $\mathrm{v}_3 = 0.1\,\mathrm{v}_F$. We found that for a smaller value of the parameter $\mathrm{v}_3 = 0.5 \cdot 10^5\, m/s$ the exciton binding energy becomes smaller (blue dotted curve). And finally, when $\mathrm{v}_3$ is almost zero, i.e. when we do not take into account the parameter $\gamma_3$ that is responsible for the azimuthal asymmetry of energy bands ( see Ref. 12 ), the absorption peak decreases and the exciton binding energy becomes much smaller (dash-dotted curve in Fig.6).

## 4. Conclusion

In the present work we develop a microscopic theory of a strong electromagnetic field interaction with bilayer graphene systems with an energy gap opened by external gates. We show that at resonant photon energy close to the energy gap and by adiabatically changing the gate potentials with time, one can produce full inversion of the electron population between valence and conduction bands near the Lifshitz energy. The proposed method resembles well known Rapid Adiabatic Passage techniques in quantum optics for population inversion in two level systems [18]. The population transfer in graphene systems can also be achieved by using traditional frequency chirped electromagnetic pulses, but the suggested method (i.e. adiabatic change of the energy gap) is more convenient for a graphene system, since there are technical difficulties with terahertz radiation manipulation. In this sense, graphene devices can in turn be used for infrared and terahertz radiation detection and frequency conversion.

We found that due to relative flatness of the bottom (top) of conduction (valence) band in multilayer graphene systems in the presence of a perpendicular electric field, the density of coherently created particle-hole pairs becomes quite large, which can make Bose-Einstein condensation of electron-hole pairs possible.

We consider excitonic states in monolayer and bilayer graphene systems with opened energy gap. We study the dependences of the exciton binding energy on different parameters describing the band structure of graphene system.

To take into account the Coulomb interaction, we use Hartree-Fock approximation that leads to a closed set of equations for the single-particle density matrix, which in turn produces our final results for the excitonic absorption in this case. The developed method can be used for investigation of excitonic states in more general multilayer graphene systems.


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