EXISTENCE OF MAXIMUM ENTROPY PROBLEM SOLUTION IN A GENERAL N-DIMENSIONAL CASE

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Maximum entropy methodology applied in European call options seeks a risk neutral probability measure p, such that

$$Ap = b \tag{1}$$

$$\sum_{i=1}^{n} p_i = 1, \quad p_i \geq 0 \tag{2}$$

$$S(p) = -\sum_{i=1}^{n} p_i ln(p_i) \quad \text{is maximal} \tag{3}$$

where b is the vector of current option prices' future values for each strike and A is the matrix of pay-offs. We denote A's columns by $a_0, a_1, \ldots, a_n$ (note that $a_n = a_{n-1} + tI$ for some t). Consider the following $n+1$ hyperplane - vector pairs (we denote hyperplanes by $hp(\cdot)$ and convex hulls by $conv(\cdot)$).

$$\begin{cases} hp(a_1, a_2, \ldots, a_{n-1}, I), & a_0 \\ \vdots \\ hp(a_0, a_1, \ldots, a_{n-2}, a_{n-1}), & I \end{cases} \tag{4}$$

For each hyperplane we denote by N_i its normal "pointing" in the direction of the associated vector a_i. It is obvious that there exists a finite t, s.t. 1, 2 are satisfied if and only if the following inequalities take place.

$$\begin{cases} \langle N_0 - a_1, b - a_1 \rangle \geq 0 \\ \vdots \\ \langle N_n, b \rangle \geq 0 \end{cases} \tag{5}$$

Assuming that condition 5 is true, the following lemmas and the corresponding theorem are true.

Lemma 1. *$\exists \mu > 0$, s.t. $\forall t$ for which $b \in conv(a_0, \ldots, a_{n-1}, a_n)$, $t \geq \mu > 0$.*

Lemma 2. *If for some t_0 $b \in conv(a_0, \ldots, a_n)$, then this also holds for any $t > t_0$.*

Lemma 3. *Let T be the set of all t's, s.t. $b \in conv(a_0, \ldots, a_n)$, then $\underline{t} = \inf T \in T$.*

Theorem 1. *If condition 5 is satisfied, the angle between b and I isn't 0 and $b_n > 0$, then $b \in conv(a_0, \ldots, a_n)$, where $a_n = a_{n-1} + \underline{t}I$ and the factor of a_{n-1} is 0 in the linear representation of b by vectors $a_0, \ldots, a_n$. The minimal value of t, $\underline{t}$ is given by*

$$\underline{t} = \frac{b_n(K_n - K_{n-1})}{b_{n-1} - b_n} \tag{6}$$

References

[1] Alhassid Y., Agmon N., Levine R.D., *An Upper Bound for the Entropy and Its Applications to the Maximal Entropy Problem*, Chem. Phys. Lett., **53**, 1978, pp. 22.

[2] Alhassid Y., Agmon N., Levine R.D., *An Algorithm for Finding the Distribution of Maximal Entropy*, Journal of Computational Physics, **30**, 1979, 250–258.

[3] Margaryan N.D., *An Algorithmic Approach to Solving the Maximum Entropy Problem*, Proc. of Engineering Academy of Armenia, **14** (3), 2017, 371–374.

[4] Margaryan N.D., *A Boundary for the Existence of Solution to the Maximum Entropy Problem Applied in European Call Options*, Proc. of the Yerevan State University, **52** (1), 2018, 3–7.