QUANTUM ENTANGLEMENT IN TWO COUPLED KERR-NONLINEAR RESONATORS

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Introduction

Quantum computers and quantum computing function on the basis of algebra of quantum logic. The "if ..., to ..." argument in quantum logic is a reversible binary operation that is physically realized only by the conditional dynamics of the coherently correlated combined system of qubits. Such dynamics does not have a classical analog and it is performed exclusively by entangled quantum states. Such states are obtained by various physical systems that must ensure the reliability and stability of quantum computing, fights with errors and decoherence due to dissipation dephasing during interaction with the environment or with imperfect control of computer equipment [1].

Entanglement is an important physical resource that underlies the quantum information protocols, including quantum cryptography [3] and teleportation [4]. For any quantum algorithm operating on pure states, the presence of multiparticle (multiple-qubit) entanglement is necessary for exponential acceleration by classical calculations [5]. Therefore, the ability to control entangled states is one of the basic requirements for building quantum information systems.

The Hamiltonian of the combined systems of qubits A and B should have the form:

\[ H = H_A + H_B + H_{AB} + H_C(t). \]  

Here \( H_A \) and \( H_B \) are the Hamiltonians of subsystems A and B respectively, \( H_{AB} \) is the interaction energy of qubits A and B, which is responsible for the existence of entanglement, \( H_C(t) \) is the energy of the control signal responsible for transitions between different states. To have such a Hamiltonian for obtaining quantum entangled states, we will consider two coupled Kerr-nonlinear resonators and calculate the probability of these states.

Quantum entanglement and violation of inequality for two nonlinear resonators

Let us consider two Kerr-nonlinear resonators interacting with one another under pulsed excitation. The Hamiltonian describing our system in the approximation of a rotating wave has the form:

\[ H = \Delta_1 a_1^+ a_1 + \Delta_2 a_2^+ a_2 + \chi_1 (a_1^+)^2 a_1^2 + \chi_2 (a_2^+)^2 a_2^2 + g (a_2^+ a_1 + a_1^+ a_2) + \Omega f(t) (\Omega a^+ + \Omega^* a). \]  

Here the coupling constant, depending on the time \( \Omega f(t) \), which is proportional to the amplitude of the leading field \( \Omega \), consists of Gaussian pulses of duration \( T \) separated by time intervals \( \tau \),

\[ f(t) = \sum e^{-(t-t_n-\tau)^2/\tau^2}. \]

\( a_j \) Bose annihilation operators associated with modes, \( g \) is the Jaynes-Cummings connection constant, which we consider real without substantial loss of generality. \( \chi_1, \chi_2 \) nonlinearity forces in each oscillator, \( \Delta_1 = \omega_{11} - \omega \), \( \Delta_2 = \omega_{12} - \omega \) is the detuning between the average frequency of the exciting field and the frequency for each oscillator. This model is represented experimentally feasible and can be realized in several physical systems. The quantum optical behavior of our system is completely described by the master equation for the density matrix

\[ \frac{d\rho}{dt} = [H, \rho] + \frac{\Gamma}{2} \sum_{n=1}^{2} \left( 2 a_n \rho a_n^+ - a_n^+ a_n \rho - \frac{1}{2} \rho a_n^+ a_n \right). \]

The second and third terms on the right-hand side of this equation are Lindblad members, necessary for describing the Markov connection between the system and the random environment.
They describe, respectively, dissipation and pure dephasing of processes [6]. Here \( n_j = a_j^\dagger a_j \) the filling operator of the \( j \) mode (the number of photons). Equation (3) can be solved by expanding the density matrix over the basis of the number of particles in the same way as in [16].

Our goal is to demonstrate the continuous variable entanglement between the modes in the first and the third quantum boxes and to calculate the probability of states distribution \(|00\rangle, |10\rangle, |01\rangle \) and \(|11\rangle\). By analogy with Bell's result for discrete variable entanglement, continuous variable entanglement is characterized by a violation of the inequality [7, 8]:

By analogy with Bell's result for a discrete variable entanglement, the CV-interlacing between the modes \( n \) and \( m \) is characterized by a violation of the inequality [9, 10]:

\[
1 \leq S_{nm} = V(\hat{p}_n - \hat{p}_m) + V(\hat{a}_n - \hat{a}_m),
\]

(4)

where the amplitude and phase operators are defined as

\[
\hat{p}_n = (\hat{a}_n + \hat{a}_n^\dagger)/2 \quad \text{and} \quad \hat{a}_n = (\hat{a}_n - \hat{a}_n^\dagger)/(2i).
\]

The variance of the operator is defined by

\[
V(\hat{O}) = \hat{O}^2 - \langle \hat{O}^2 \rangle.
\]

To be sure that inequality (4) indicates entanglement, if it is present, it may be necessary to minimize the value of \( S_{12} \) regarding to the phase counts of different modes. Local operations allow to make transformation \( \hat{a}_n \rightarrow \hat{a}_n e^{-i\phi_n} \) and the minimum value of \( S_{nm} \), obtained when \( \phi_m + \phi_n \) is an integral multiple of \( 2\pi \), is given by formula

\[
S_{nm} = 1 + \hat{a}_n^\dagger \hat{a}_n + \hat{a}_m^\dagger \hat{a}_m - \hat{a}_n^\dagger \hat{a}_m - \hat{a}_m^\dagger \hat{a}_n - 2\sqrt{\hat{a}_n^\dagger \hat{a}_m^\dagger \hat{a}_m \hat{a}_n}.
\]

(5)

According to calculations based on the von Neumann equations [6] the WFMC (wave function Monte Carlo) approach demonstrates cohesion between the first and second modes in the system via violation of the inequality \( 1 \leq S_{12} \). We have solved the basic equation Eq. (5) and obtained dependence of \( S_{12} \) value from time, which is shown in Fig. 6.

**Probability distribution of entanglement states**

We solve the master equation Eq. (3) numerically based on quantum state diffusion method. The applications of this method for NDO studies can be found in [29]-[32].

In Figs. 1-4 the solid (A) lines show the probability of state in the first resonator, dotted (B) lines in second one. Considering the pulsed regimes of Kerr-nonlinear reservoir, we assume that the spectral pulse-width, i.e. the spectral width of pulses, should be smaller than the nonlinear shifts of the oscillatory energy levels. It means that the duration of pulses should be greater than \( \frac{1}{\chi} \).
Thus, for strong nonlinearities $\chi/\gamma > 1$, we arrive at the following inequalities for the duration of Gaussian pulses $1/\gamma > T > 1/\chi$. The parameters are: $\chi/\gamma = 15$, the maximum amplitude of pump field $\Omega/\gamma = 6$, $\tau = 5.5\gamma^{-1}$, $T = 0.4\gamma^{-1}$, $g = 3$.

Fig. 1. Quantum entanglement state $|00\rangle$ probability for entanglement qubits A and B.

Fig. 2. Quantum entanglement state $|01\rangle$ probability for entanglement qubits A and B.

Fig. 3. Quantum entanglement state $|10\rangle$ probability for entanglement qubits A and B.
In this work, we have calculated entanglement between two modes of coupled Kerr-nonlinear resonators which satisfies the condition of quantum entanglement. We have also calculated the probability of the state of states $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$. These states can be used in quantum computer algorithms.

References