ELASTIC STRAIN DISTRIBUTION IN
ONE LAYER QUANTUM RING SUPERLATTICE

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The elastic strain distribution in one layer InAs/GaAs quantum ring superlattice is calculated using the Green function method. The dependencies of the strain tensor components on spatial coordinate in three different directions are obtained. The comparison of the obtained results with ones for a single quantum ring shows that the effect of the matrix material on the strain distribution inside the ring is more pronounced in the case of single quantum ring. The behavior of the strain tensor components in different directions is strongly different both for the single quantum ring and quantum ring superlattice due to the anisotropy of the crystal lattice.

Keywords: quantum ring superlattice, elastic strain, Green function method.

Introduction. Semiconductor heterostructures containing zero-dimensional objects, quantum dots (QD) and quantum rings (QR), have a wide range of potential applications as new or improved active elements in a number of optoelectronic devices. In most cases, these structures are fabricated with an intrinsic elastic strain field arising from the lattice mismatch between the QD and matrix materials. Knowledge of this strain field is crucial for further device modeling since the strain strongly effects on the performance of optoelectronic devices [1–7].

Recently impressive progress has been made in the field of manufacturing of ordered structures composed of two or three dimensional arrays of QRs [8–10].

In this work we have calculated the components of the strain tensor for one layer superlattice (SL) composed of InAs/GaAs QRs. The misfit strain for these materials (about 6.7%) leads to a considerable strain distribution [11], which may significantly effect on the SL band structure.

Theory. Implying the Green function method suggested by Andreev et. al [3], and taking into account the lattice cubic symmetry we obtain the diagonal components of the strain tensor in one layer quantum ring superlattice (QRSL) with lattice constants $a_x$ and $a_y$:

$$E_{ii} = \epsilon_0 \sum_{\vec{R}} \chi(\vec{r} - \vec{R}) - \frac{2\epsilon_0(C_{11} + 2C_{12})}{S_0} \times$$

$$\sum_{\xi} \frac{\exp(i\xi_\perp)}{\xi_\perp} \int_{-\infty}^{\infty} \frac{\xi_\perp^2 \exp(i\xi_\perp r) \sin(h\xi_\perp/2)F(\xi_\perp)}{\xi_\perp(C_{44}\xi_\perp^2 + C_{44n}\xi_\perp^2) \left[1 + (C_{12} + C_{44})\sum_{j=1}^{\infty} \frac{\xi_\parallel^2}{(C_{44}\xi_\parallel^2 + C_{44n}\xi_\parallel^2)} \right]} d\xi_\parallel, \quad (1)$$

where

$$F(\xi_\perp) = R_2 J_1(R_2\xi_\perp) - R_1 J_1(R_1\xi_\perp). \quad (2)$$

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In Eq. (1) \( \chi^S(\vec{r}) \) is the shape function of single QR (which is unity inside the ring and is zero outside it) with inner and outer radii \( R_1 \) and \( R_2 \) respectively, and with height \( h \), \( J_i(\xi) \) is the first kind Bessel function of the \( i \)-th order, \( C_{11}, C_{12} \), and \( C_{44} \) are the elastic constants, \( C_{an} = C_{11} - C_{12} - 2C_{44} \) is the parameter of anisotropy, \( \xi_\perp \) and \( \xi_\parallel \) are the components of the reciprocal space vector perpendicular and parallel to the QR axis respectively, \( S_0 = a_{GaAs} \) is the area of the QRSL unite cell, \( \varepsilon_0 = \frac{a_{GaAs} - a_{InAs}}{a_{InAs}} \approx -0.067 \) is the initial strain due to the lattice mismatch between the materials of QR and the surrounded matrix. The summations in (1) is carried out by the Cartesian components of the reciprocal space vector.

**Results and Discussion.** The numerical calculations are made for following values of parameters: \( R_1 = 3, R_2 = 10, h = 2, a_x = a_y = 22 \text{ nm} \). In Figure the diagonal components of the strain tensor as functions of coordinate in three different directions (100), (210) and (110) are presented.

The dotted curves represent the strain components in single quantum ring (SQR) at the plane \( z = 0 \). First of all it can be seen that in the cases of \( z = 0 \) and \( z = 0.5 \text{ nm} \) there are abrupt shifts in the strain at the inner and outer edges of the QR in contrast with the case of \( z = 1.2 \text{ nm} \) (the dash-dotted curve) because the plane \( z = 1.2 \text{ nm} \) does not crosses the QRs. It is obvious that in the case of SQR the strain component merges to zero for enough large values of the coordinate. The lattice constant of the ring material (about 0.606 nm) is larger than the lattice constant of the matrix (0.563 nm). In result the tensile strain \( E_{zz} \) (see Figs. g, h, i) increases when approaching to the ring’s inner edge from the left or to the outer one from the right. Inside the QR the initial strain decreases when the value of the coordinate approaches to the values of the inner or the outer radii. The behaviors of the curves obtained for the strain component \( E_{xx} \) (Figs. a, b, c) can be explained in a similar manner as in the previous
case. However, in contrast to the previous case, there is a qualitative difference between the dependence in (110) direction and two others. Namely, the compressive strain in the ring region in (110) direction is larger than outside the ring due to the stronger influence of the matrix material in this direction. It is obvious from Fig. c and f, that in the direction (110) the $E_{yy}$ coincides with $E_{xx}$ due to the system symmetry. Comparing Figs. d, e and f one can conclude, that the tensile strain in the (100) direction for $r > R_2$ transfers to a compressive one in the (110) direction due to the material anisotropy. It should be mentioned that our results are in qualitative agreement with the results obtained in [11] for quantum disk structures using three different methods: the isotropic elasticity theory, anisotropic continuum model and atomistic calculations.

**Conclusion.** The Green function method for anisotropic continuum model has been implied for calculation of strain distribution in one layer QRSL. The obtained results are in qualitative agreement with the results obtained in [11] for quantum disk structures using three different methods: the isotropic elasticity theory, anisotropic continuum model and atomistic calculations. A considerable differences between the dependencies of the strain components on coordinate in (100), (210) and (110) directions are observed not only for SL, but also for SQR, because of the material anisotropy.

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**REFERENCES**