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Formation of Planetesimals by Burgers Vortex

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Introduction

The radial dependence of IR, sub mm and cm radiation of protoplanetary disks (PPD) analyses show that vortices serve as incubators for growth dust particles and formation of planetesimals. Formation of planetesimals from micron size dust grains possibly, involves many physical processes. The initial stage of growth probably proceeds through the nucleation of sub-micron-sized dust grains from the primordial nebula, which then forms the monomers of fractal dust aggregates up to ~ 1mm to ~ 10cm for characteristic time of an order of 10^3 years. In this stage the dynamics of particles and coagulation are regulated by van der Waals forces and the Brownian motion.

Rigid particles, under the action of a head wind drags, lose the angular momentum and energy. As a result, the $\sim$10 cm to meter-sized particles drift to the central star for hundreds of years.

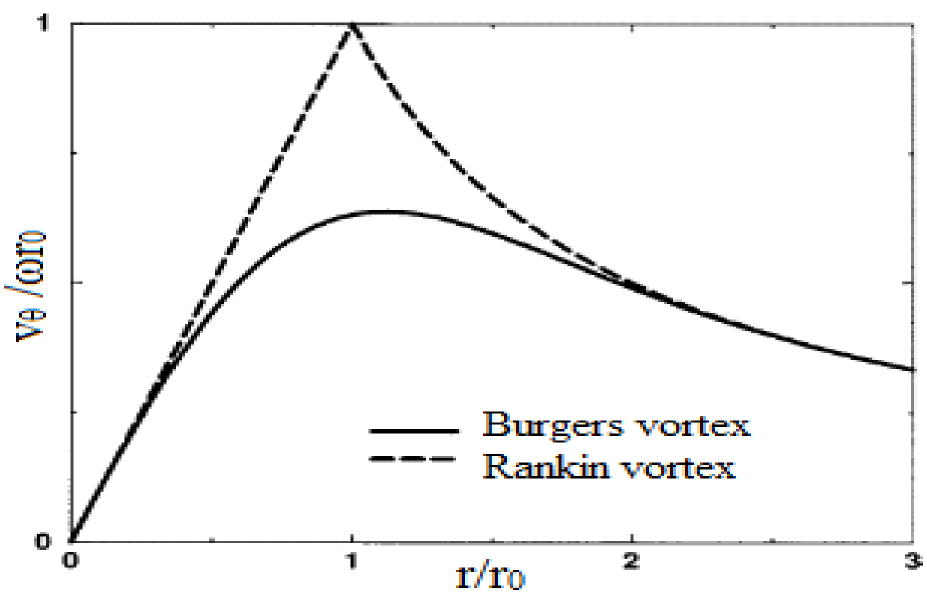
Long-lived vortical structures in gas disk are a possible way to concentrate the $\sim$10 cm to meter sized particles and to grow up them in planetesimal.

In some areas of the stratified protoplanetary disks the current has 2D-turbulent character. In laboratory experiments, formation of Burgers vortex which will be considered in the present work is often observed in 2D-turbulent flows. Anticyclonic vortices in a protoplanetary disk merge with each other and amplify, while cyclonic ones are destroyed by a shear flow.

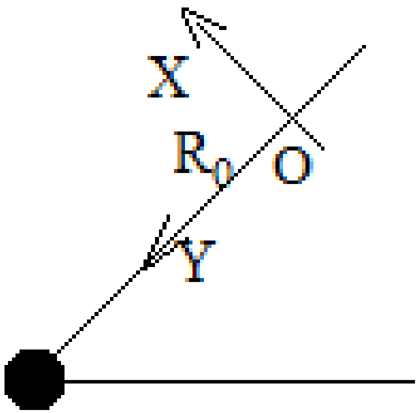
In an anticyclone, rigid particles are grasped by force of Coriolis directed to the center of a vortex. If the vortex survives ~100 rotations in a nebulae with solar mass, the quantity of the grasped particles can reach masses of planets.

The vortex of Burgers in cylindrical system of co-ordinates (r, θ, z) is defined as

$$v_r = - Ar,\ v_\theta = \omega r_0^2\ [1- \exp(-r^2/r_0^2)]/r,\ v_z = 2Az.$$



The Burgers Vortex in Local Frame of Reference



Let's use local approach, choosing frame of reference, rotating with a disk with angular speed Ω_0 at distance R_0 round the central star of mass M. In this approach, assuming the sizes of a vortex are much smaller than the distance R_0, we will choose Cartesian system of co-ordinates with center O.

In local approach, the equation stationary isentropic shear flow of gas taking into account viscosity will be described by Navier-Stokes and continuity equations

$$(\mathbf{v}\boldsymbol{\nabla})\mathbf{v} = \mathbf{j}3\Omega_0^2 y - \mathbf{k}\Omega_0^2 z - 2\boldsymbol{\Omega}_\mathbf{0}\times\mathbf{v} - \boldsymbol{\nabla} h + \nu\Delta\mathbf{v}, \qquad \nabla(\rho\mathbf{v}) = 0,$$

where $h = \int\rho^{-1}dp$ is specific enthalpy.

In the chosen Cartesian co-ordinate system the Burgers vortex will be presented in the form

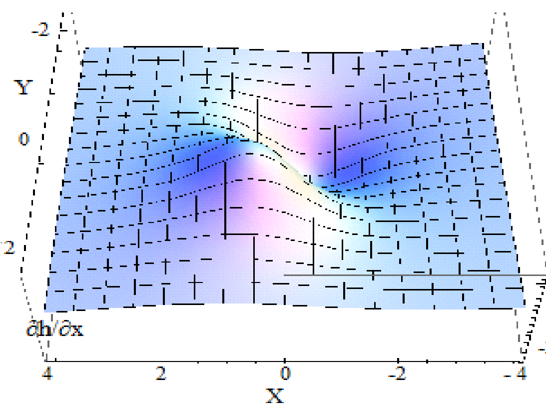
$$v_x = - Ax - \omega r_0^2 y\ [1- \exp(-r^2/r_0^2)]/r^2,$$

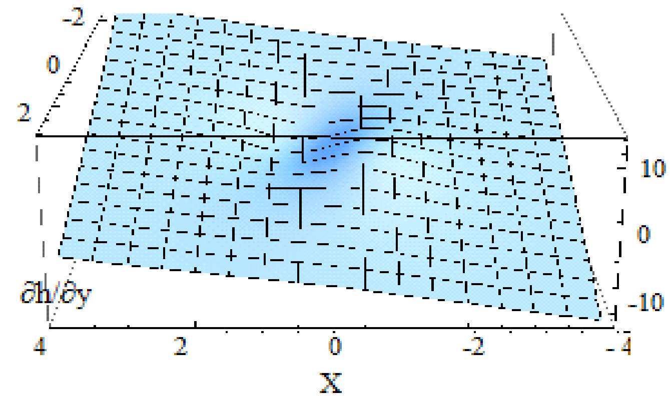
$$v_y = - Ay + \omega r_0^2 x\ [1- \exp(-r^2/r_0^2)]/r^2,$$

$$v_z = 2Az,$$

where $r^2 \equiv x^2 + y^2$.

Profile of pressure gradient force ∇h





The Dynamics of Rigid Particles in Burgers Vortex

We will study two-dimensional dynamics of dust rigid particles in a Burgers vortex taking into account action pressure gradient force ∇h, tidal force, forces of Coriolis and Stokes drag force

$$\boldsymbol{f} = \beta(\mathbf{v} - \mathbf{u}), \text{ where } \beta \equiv 18\, \rho\nu/\rho^* D^2,$$

$\mathbf{u} = (dX/dt, dY/dt)$ is velocity of a particle.

Equations governing the dynamics of rigid particles are

$$du_x/dt = 2u_y + \gamma\,(v_x|_{r = (X, Y)} - u_x) - \partial h/\partial x|_{r = (X, Y)},$$

$$du_y/dt = 3y - 2u_x + \gamma\,(v_y|_{r = (X, Y)} - u_y) - \partial h/\partial y|_{r = (X, Y)},$$

where γ is dimensionless parameter

$$\gamma = \beta/\Omega_0 = 18\, \rho\nu/\rho^* D^2 \Omega_0$$

The dynamics of particles in the vortex trunk

The equations of motion of rigid particles in matrix form looks like

$$\begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{u}_x \\ \dot{u}_y \end{Bmatrix} = \begin{Bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a & b & -\gamma & 2 \\ -b & a & -2 & -\gamma \end{Bmatrix} \begin{Bmatrix} X \\ Y \\ u_x \\ u_y \end{Bmatrix};$$

where $a = A(A - \gamma) - (\omega + 1)^2 + 1$; $b = 2A(\omega + 1) - \gamma\omega$.

From these equations it is visible that equilibrium position of rigid particles in a vortex trunk is its center X = Y = 0, where $u_x = u_y = 0$ and $\dot{u}_x = \dot{u}_y = 0$.

For the stability of this position of balance, it is necessary the real parts of eigenvalues of a matrix to be zero or negative. This leads to stability criterion

$$\gamma > \mathrm{A},$$

which in a dimensional form, for ν gives

$$\nu > \rho^* A D^2 / 18\rho.$$

Further study shows there is not exist any anticyclonic orbit in the volume of Burgers vortex on which the sum of Coriolis and drag forces is counterbalanced by centrifugal force, Hence, unique position of balance for rigid particles in a Burgers vortex is its center where all particles captured by a vortex will gather during the characteristic time

$$\tau \sim \omega r_{eff}/A\sqrt{\beta\nu}.$$

The mass of the rigid particles captured by a vortex is an order

$$M_p \approx \pi\, r_{eff}^2\, \Sigma^*$$

For typical PPD estimations of τ and $M_{p\ give}$

$$M_p \approx 10^{28}\text{g};\ \tau \sim 3\cdot 10^6\ (\text{m/D})\ \text{yrs}$$