The 6th Gdańsk Workshop on Graph Theory

July 1-4, 2018

Gdańsk, Poland

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INTERVAL EDGE COLORINGS - RECENT RESULTS AND NEW DIRECTIONS

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An *interval coloring* of a graph G is a proper edge coloring of G such that the colors incident to any vertex of G form an interval of integers. This concept was introduced by Asratian and Kamalian in 1987. There are trivial examples of graphs, such as odd cycles, that do not have interval colorings. The smallest bipartite graph (in terms of maximum degree) with no interval coloring has maximum degree 11 and 20 vertices [7]. Bipartite graphs with maximum degree at most three always have interval colorings, the cases $4 \leq \Delta(G) \leq 10$ are open, and the general question of deciding interval colorability for bipartite graphs is NP-complete, as proved by Sevastjanov (1990). Nevertheless, it is known that trees, and regular and complete bipartite graphs always admit interval colorings.

In this talk I shall discuss some recent progress on interval colorings, and also point to some new directions in this field of research. Most results will be related to the following conjecture, which first appeared in the Master thesis of Hansen in 1992 (completed under the supervision of Bjarne Toft). An (a, b)-*biregular graph* is a bipartite graph where all vertices in one part have degree a and all vertices in the other part have degree b.

Conjecture 1. *[6] Every (a, b)-biregular graph has an interval coloring.*

Using Petersen's 2-factor theorem, Hansen deduced that all $(2, b)$-biregular graphs have interval colorings for even b; the case of odd b was settled by Hanson, Loten, Toft [5] (and independently by Konstochka, and Kamalian and Mirumian). However, even the cases $(a, b) = (3, 4)$ and $(a, b) = (3, 5)$ of Conjecture 1 are open, while the case $(a, b) = (3, 6)$ was recently settled by Casselgren and Toft [3].

A variant of interval edge coloring is obtained by considering the minimum and maximum colors in a proper edge coloring as consecutive. This model is known as *cyclic interval coloring* and was first considered by de Werra and Solot in 1991. Any graph with an interval coloring admits a cyclic interval Δ-coloring by taking all colors modulo the maximum degree Δ. Thus the following is a weakening of Conjecture 1.

Conjecture 2. *(Casselgren-Toft 2015) Every* (a, b)*-biregular graph has a cyclic interval* $\max\{a, b\}$*-coloring.*

Note that by König's edge coloring theorem, all $(k-1, k)$-biregular graphs have cyclic interval colorings. Casselgren and Toft settled the case $(a, b) = (4, 8)$ of this conjecture in the affirmative [3], and quite recently the case $(a, b) = (2k - 2, 2k)$ was confirmed by Asratian, Casselgren and Petrosyan [2] using Petersen's 2-factor theorem. Moreover, the case of $(3, 5)$-biregular and $(4, 7)$-biregular graphs have also recently been settled in the affirmative [2, 4] (albeit using 6 and 8 colors, respectively, instead of the conjectured 5 and 7).

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GRAPH SEARCHING GAMES AND PROBABILISTIC METHODS

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The application of probabilistic methods to graph searching problems such as the game of Cops and Robbers and Firefighting is a new topic within graph theory. Research on this topic emerged only over the last few years, and as such, it represents a rapidly evolving and dynamic area. Probability enters the picture in three different ways. During the talk, I will show one simple example from each class.

1) Graph searching games can be played on random graphs; as an example I will use the firefighter problem [1, 2].

2) Probabilistic methods can be used to prove results about deterministic games; as an example I will consider the robber falling to the bottom of the hypercube [3].

3) One of the players can make random moves; as an example I will use the problem of zombies and survivors [4].

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DYNAMIC MONOPOLIES AND VACCINATION

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A widely studied model for influence diffusion in social networks are *dynamic monopolies*. For a graph G and an integer-valued threshold function τ on its vertex set, a *dynamic monopoly* is a set of vertices of G such that iteratively adding to it vertices u of G that have at least $\tau(u)$ neighbors in it eventually yields the entire vertex set of G. In this talk we present recent bounds, algorithms, and hardness results for dynamic monopolies and related vaccination problems.

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RAINBOW CONNECTIONS IN DIGRAPHS

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A path P in an arc-coloured digraph is *rainbow* if no two arcs of P are coloured with the same colour. A graph is *rainbow connected* if any two vertices are connected by a rainbow path. A digraph is *strongly rainbow connected* if for every pair of vertices (u, v) there exists a shortest path from u to v that is rainbow.

A path P in a vertex-coloured digraph is *vertex rainbow* if its internal vertices have distinct colours. A digraph is *rainbow vertex-connected* if any two vertices are connected by a vertex rainbow path. A digraph is *strongly rainbow vertex-connected* if for every pair of vertices (u, v) there exists a shortest path from u to v that is vertex rainbow.

A path P in a totally-coloured digraph is *total rainbow* if its edges and internal vertices have distinct colours. A digraph is *total rainbow connected* if any two vertices are connected by a total rainbow path. A digraph is *strongly rainbow connected* if for every pair of vertices (u, v) there exists a shortest path from u to v that is total rainbow.

The *rainbow connection number* (*rainbow vertex-connection number*, *total rainbow connection number*) of a strong digraph D, is the minimum number of colours needed to make the digraph rainbow connected (rainbow vertex-connected, total rainbow connected). The rainbow connection number, the rainbow vertex-connection number, the total rainbow connection number are denoted by $\overrightarrow{\mathrm{rc}}(D)$, $\overrightarrow{\mathrm{rvc}}(D)$ and $\overrightarrow{\mathrm{trc}}(D)$, respectively.

The *strong rainbow connection number* (*strong rainbow vertex-connection number*, *total strong rainbow connection number*) of a strong digraph D is the minimum number of colours needed to make the digraph strongly rainbow connected (strongly rainbow vertex-connected, total strongly rainbow connected). The strong rainbow connection number, the strong rainbow vertex-connection number, the total strong rainbow connection number are denoted by $\overrightarrow{\mathrm{src}}(D)$, $\overrightarrow{\mathrm{srvc}}(D)$ and $\overrightarrow{\mathrm{strc}}(D)$, respectively.

In this talk, we consider rainbow connection numbers. We give some properties of these numbers and establish relations between them. The rainbow connection number and the rainbow vertex-connection number of a digraph D are both upper bounded by the order of D, while its total rainbow connection number is upper bounded by twice of its order. In particular, we characterize digraphs of order n with rainbow connection number n, rainbow vertex-connection number n, and total rainbow connection number $2n$, respectively. We consider the strong rainbow connection number of minimally strongly connected digraphs and non-Hamiltonian strong digraphs. Furthermore, we present an overview of known results for special classes of digraphs.

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EDGE COLORINGS WITH CONSTRAINTS

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A strong edge-coloring is a proper edge-coloring in which the edges of every color class induce a matching, i.e., edges at distance at most 2 are colored distinctly. It was conjectured by Erdős and Nešetřil (published in [1]) that $\frac{5}{4}\Delta(G)^2$ colors suffice to color any graph G with maximum degree $\Delta(G)$. This conjecture received a lot of attention and it is still widely open. However many particular results have been proved.

Apart from that, a number of similar edge-colorings have also been introduced during years. In particular, we will focus on star edge-coloring and semistrong edge-coloring. The former, introduced in [3], is a proper edge-coloring without bichromatic paths and cycles of length 4, while the latter, introduced in [2], is a proper edge-coloring in which the edges of every color class induce a semistrong matching. Here, a matching M of a graph G is semistrong if every edge of M has an endvertex of degree one in the induced subgraph $G[M]$. We will present some results on the above mentioned topics together with some techniques used in our proofs.

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DISTANCE-BASED TOPOLOGICAL POLYNOMIALS OF ZERO DIVISOR GRAPHS OF FINITE RINGS

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The applications of finite commutative ring as useful substances in robotics and automatic geometric, information and communication theory, elliptic curve cryptography, physics and statistics. Let $G(V, E)$ be a simple and connected graph, the distance between two distinct vertices $u, v \in V(G)$ is the number edges in the shortest path between them, it is denoted by $d(u, v)$. The number of edges in the longest distance in G is called the diameter of the graph G, $D(G)$. The set of all neighbors of a vertex u of G is called the neighbourhood of u and the cardinality of the neighbourhood of u is the degree of the vertex u, denoted as d_u.

A real valued function $\phi : G \to \mathbb{R}$ which maps each structure to certain real numbers is known as topological index. In this article, by the help of graphical structure analysis, we investigate a few distance-based topological polynomials and indices of zero divisor graphs of finite rings.

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RANDOM GENERATION AND RELATED ALGORITHMS FOR SEMI-FIBONACCI TREES

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Fibonacci trees is a beautiful class of binary search trees which represents fully unbalanced AVL trees that in every branch, the height of the left subtree is bigger than the height of the right one. We define a *Fibonacci-isomorphic* tree as an ordered tree which is isomorphic to a Fibonacci tree and Isomorphism on rooted trees is defined in [1]. Note that two AVL trees (generally, ordered trees) are isomorphic iff there exists a one-to-one correspondence between their nodes that preserves not only adjacency relations in the trees, but also the roots.

Figure 1 shows two Fibonacci-isomorphic trees of height 5, the left one is a Fibonacci tree of height 5 and the right one is one of its isomorphisms.

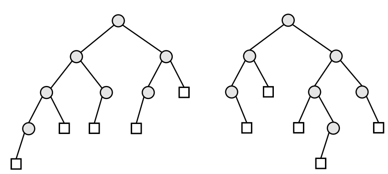

Figure 1: Two isomorphic Fibonacci trees.

We are interested to study the random generation and other combinatorial algorithms of this class of trees. This work is partially related to the work published in [2].

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ON IRREGULAR LABELINGS OF GRAPHS

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Chartrand *et al.* in [2] introduced edge k-labeling ϕ of a graph G such that $w_\phi(x) \neq w_\phi(y)$ for all vertices $x, y \in V(G)$ with $x \neq y$, where weight of a vertex $x \in V(G)$ is $w_\phi(x) = \sum \phi(xy)$ and the sum is over all vertices y adjacent to x. Such labelings were called *irregular assignments* and the *irregularity strength* $s(G)$ of a graph G is known as the minimum k for which G has an irregular assignment using labels at most k. In 2007, Bača *et al.* in [3] started to investigate two modifications of the irregularity strength of graphs, namely a *total edge irregularity strength*, denoted by $tes(G)$, and a *total vertex irregularity strength*, denoted by $tvs(G)$.

Motivated by these irregular labeling, Ahmad *et al.* in [1] introduced vertex k-labeling $\phi : V \to \{1, 2, \ldots, k\}$ that can be defined as *edge irregular k-labeling* of the graph G if for every two different edges e and f there is $w_\phi(e) \neq w_\phi(f)$, where the weight of an edge $e = xy \in E(G)$ is $w_\phi(xy) = \phi(x) + \phi(y)$. The minimum k for which the graph G has an edge irregular k-labeling is called the *edge irregularity strength* of G, denoted by $es(G)$.

Algorithms help in solving many problems, where other mathematical solutions are very complex or impossible. Graph algorithms are heavily used in different scientific fields. Similarly computational methods has helped in tackling numerous issues where other numerical arrangements are extremely perplexing or incomprehensible. In this paper, the edge irregularity strength is discussed using the algorithmic approach for certain graphs whose edge irregularity strength is difficult to calculate by conventional method.

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GRAPH INTERPRETATIONS OF THE FIBONACCI NUMBERS

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In the talk we present a new generalization of the Fibonacci numbers and we give their graph interpretation.

Let $k \geqslant 2, n \geqslant 0$ be integers and let $p \geqslant 1$ be a rational number. The **p-generalized Fibonacci numbers** $F_{k,p}(n)$ are defined recursively in the following way

$$F_{k,p}(n) = pF_{k,p}(n-1) + (p-1)F_{k,p}(n-k+1) + F_{k,p}(n-k) \text{ for } n \geqslant k$$

with the initial conditions

$$F_{k,p}(n) = \begin{cases} 0 & \text{for } n = 0 \\ p^{n-1} & \text{for } 0 < n \leqslant k - 1 \end{cases}.$$

We give some properties of numbers $F_{k,p}(n)$ and we show that these numbers have the graph interpretation related to the total graph interpretation of the numbers of the Fibonacci type introduced in [1].

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GENERALIZED KERNELS IN GRAPHS

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In the talk we present $(1,1,2)$-kernels and strong $(1,1,2)$-kernels in graphs being a generalization of classical kernels, $(1,2)$-kernels and 2-dominating kernels, simultaneously. We present the necessary and sufficient conditions for the existence of strong $(1,1,2)$-kernels in some classes of graphs. Moreover we consider lower and upper strong $(1,1,2)$-kernel numbers and dependencies between them.

RAMSEY NUMBERS FOR SOME SELECTED GRAPHS

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The $k-$colour Ramsey number $R(G_1, G_2, ..., G_k)$ is the smallest integer n such that in arbitrary edge k-colouring of K_n a subgraph G_i in the colour $i, 1 \leq i \leq k$ is contained. The Turán number $ex(n, G)$ is the maximum number of edges of a graph on n vertices which does not contain G as a subgraph. We study $R(G_1, G_2, ..., G_k, C_m)$, where G_i $(1 \leq i \leq k)$ is a linear forest with small components and C_m is a cycle of order m, where $m \geq 3$. We generalize some result published in [1–3]. We apply Turán numbers for counting the upper bounds [3-4].

There are open problems even for $k = 2$.

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GAMMA GRAPHS OF TREES

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Every dominating set of the smallest possible cardinality is called γ-set. We consider a graph $\gamma.G$, whose vertices correspond to γ-sets of G, and two γ-sets S, S' are adjacent in $\gamma.G$ if there exist such adjacent vertices $u, v \in V(G)$ that $S = S' \setminus \{u\} \cup \{v\}$ and $u \neq v$.

The results presented in this talk refer to problems presented in a paper of Fricke et al. [1] about gamma graphs of trees.

It can be shown that $\Delta(T(\gamma)) = \mathcal{O}(n)$ for any tree. Counterexamples which prove that the equality $|V(T(\gamma))| < 2^{\gamma(T)}$ is not true for any tree T will be presented. Even though, the percentage of trees for which the inequality does not hold is very small, it is natural to ask a question about a characterization. Gamma-graphs of trees for which $|V(T(\gamma))| = 2^{\gamma(T)}$ are usually isomorphic to n-dimensional cubes. That is why a characterization of a class of trees for which gamma graphs are cubes will be presented.

Keywords: dominating sets, gamma graph, maximal degree, gamma tree.

AMS Subject Classification: 05C69, 05C07.

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THE RANDIĆ ENTROPY OF GRAPHS AND RELATED INFORMATION-THEORETIC INDICES FOR GRAPHS

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In this talk, we discuss graph entropy measures and some properties thereof. The Randić entropy [1] is based on the so-called Randić weights [2]. It turns out that the Randić is highly degenerate. Also, we introduce some related information-theoretic graph measures and see that they capture structural information uniquely.

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ZERO-VISIBILITY COPS&ROBBER

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The classical Cops&Robber game is defined as follows [1, 2]. The game is played on a graph by two parties that alternate their moves. First the player that controls a number of $k \geq 1$ cops places each cop on some vertex of the graph. Then, the robber chooses its location. This provides an initial configuration and the players start performing their moves. The first player, in its turn, makes the following for each cop: the cop either stays idle or moves to a vertex adjacent to its current position. Then the robber either moves to a neighbor or does not move. The cops win if at any point of the game a cop and the robber are on the same vertex — thus the robber is captured (in such case the graph is called k-cop-win). The robber wins if it can avoid being captured indefinitely. This is a perfect information game, i.e., at any point each player can see the positions of all entities.

In this talk we discuss a version of the game in which the cops have no visibility: at any point of the game the robber has full information about the positions of the cops but the cops have no information where the robber is. Thus, they can only deduce the potential locations of the robber from the history of their moves. We survey some properties of the game, how it differs from the classical version, and list some results obtained by the authors in [3, 4].

There is a number of open problems related to zero-visibility Cops&Robber. One can see that 1-cop-win graphs are caterpillars. How k-cop-win graphs for $k > 1$ can be characterized? It is known that finding the minimum k such that an input graph is k-cop-win is NP-hard. Is there a good approximation polynomial-time algorithm? In the classical version of the game, there exist upper bounds on the length of the game (the minimum number of moves that guarantee the capture of the robber). How long the zero-visibility game needs to be for any $k \geq 1$. For a wider list of open problems see [3, 4].

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EXTREMAL COLORINGS AND INDEPENDENT SETS

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We discuss some extremal problems of maximizing the number of colorings and independent sets over families of graphs with fixed chromatic number and various different connectivity conditions.

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UNIQUELY IDENTIFYING THE EDGES OF GRAPHS

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Parameters related to distances in graphs have attracted the attention of several researchers since several years, and recently, one of them has centered several investigations, namely, the metric dimension. A vertex v of a connected graph G *distinguishes* two vertices u, w if $d(u, v) \neq d(w, v)$, where $d(x, y)$ represents the length of a shortest $x - y$ path in G. A subset of vertices S of G is a *metric generator* for G, if any pair of vertices of G is distinguished by at least one vertex of S. The minimum cardinality of any metric generator for G is the *metric dimension* of G and is denoted by $dim(G)$. This concept was introduced by Slater in [4] in connection with some location problems in graphs. On the other hand, the concept of metric dimension was independently introduced by Harary and Melter in [1].

A metric generator uniquely recognizes the vertices of a graph in order to look out how they do "behave". However, what does it happen if there are anomalous situations occurring in some edges between some vertices? Is it possible that metric generators would properly identify the edges in order to also see their behaving? The answer to this question is negative. In connection with this, the following concepts deserve to be considered.

Given a connected graph $G = (V, E)$, a vertex $v \in V$ and an edge $e = uw \in E$, the distance between the vertex v and the edge e is defined as $d_G(e, v) = \min\{d_G(u, v), d_G(w, v)\}$. A vertex $w \in V$ *distinguishes* two edges $e_1, e_2 \in E$ if $d_G(w, e_1) \neq d_G(w, e_2)$. A set $S \subset V$ is an *edge metric generator* for G if any two edges of G are distinguished by some vertex of S. The smallest cardinality of an edge metric generator for G is the *edge metric dimension* and is denoted by $edim(G)$ [2].

A kind of mixed version of these two parameters described above is of interest. That is, a vertex v of G *distinguishes* two elements (vertices or edges) x, y of G if $d_G(x, v) \neq d_G(y, v)$. Now, a set $S \subset V$ is a *mixed metric generator* if any two elements of G are distinguished by some vertex of S. The smallest cardinality of a mixed metric generator for G is the *mixed metric dimension* and is denoted by $mdim(G)$ [3].

In concordance with the concepts above, several combinatorial results of $dim(G)$, $edim(G)$ and $mdim(G)$ shall be given in this talk. Moreover, some open problems like the following ones, will also be discussed.

- There are several graphs in which no metric generator is an edge metric generator. So, we could think that probably any edge metric generator is also a standard metric generator. Nevertheless, this is further away from the reality, although there are several graph families in which such fact occurs. In this sense, an open problem concerning characterizing the graphs G for which $dim(G) = edim(G)$, $dim(G) < edim(G)$ or $dim(G) > edim(G)$ was pointed out in [2], and it is indeed already studied in other works (see [5, 6]).

- In contrast with the item above, for the case of mixed metric dimension, it clearly follows that that any mixed metric generator is also a metric generator and an edge metric generator. Thus, it immediately follows that for any graph G, $mdim(G) \geq \max\{dim(G, edim(G)\}$. Consequently, characterizing the graphs G for which $mdim(G) = edim(G)$ or $mdim(G) = dim(G)$ is of interest in the research.

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THOSE MAGNIFICENT BLIND COPS IN THEIR FLYING MACHINES WITH SONARS

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Inspired by the localisation problems in wireless networks we study the variation of Cops and Robber model [4], in which helicopter cops plays against a slow, but invisible robber. Instead, cops receive the information of a distance from robber's current position to vertices probed by the cops. The goal for cops is to localise the robber. This model, restricted to one cop, was introduced by Seager [5] (with slightly different rules) and then by Carraher, Choi, Delcourt, Erickson and West [3].

We investigate the cop number in this model providing some bounds of it, developed from other graph parameters (eg. pathwidth). We show that cop number of outerplanar graphs is at most 3, while, surprisingly, it is unbounded for planar graphs (even of treewidth 2). The talk is based on results presented in papers [1, 2].

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INDUCED RAMSEY NUMBERS INVOLVING MATCHINGS

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We say that a graph F *strongly arrows* a pair of graphs (G, H) and write $F \stackrel{\text{ind}}{\longrightarrow} (G, H)$ if any colouring of its edges with red and blue leads to either a red G or a blue H appearing as induced subgraphs of F. *The induced Ramsey number*, $IR(G, H)$ is defined as $\min\{|V(F)| : F \stackrel{\text{ind}}{\longrightarrow} (G, H)\}$. Obviously $IR(G, K_2) = |V(G)|$. Moreover $tG \stackrel{\text{ind}}{\longrightarrow} (G, tK_2)$, where tG denotes the graph that is a pairwise vertex-disjoint union of t copies of G. This implies that $IR(G, tK_2) \leq t|V(G)|$. We show that this inequality may be strict. On the other hand, we provide some cases for which it holds as equality.

ON A CLASS OF GRAPHS WITH LARGE TOTAL DOMINATION NUMBER

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Let $G = (V(G), E(G))$ be a simple graph. A set $S \subseteq V(G)$ is called a *dominating set* of G if every vertex of $V(G) \setminus S$ is adjacent to a member of S. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. If G has no isolated vertices, a subset $S \subseteq V(G)$ is called a *total dominating set* of G if every vertex of $V(G)$ is adjacent to a member of S. The *total domination number* $\gamma_t(G)$ of a graph G with no isolated vertices is the minimum size of a total dominating set of G.

It is well-known that $\gamma_t(G) \leq 2\gamma(G)$. We provide a characterization of a large family of graphs (including chordal graphs) satisfying $\gamma_t(G) = 2\gamma(G)$, strictly generalizing the results of Henning [1] and Hou et.al. [2], and partially answering an open question of Henning [3]. Furthermore, our characterization also yields a polynomial-time recognition algorithm for such graphs.

The neighborhood $N(v)$ of a vertex v is the set of vertices adjacent to v. The closed neighborhood $N[v]$ of v is $N(v) \cup \{v\}$. Two vertices $u, v \in V(G)$ are *true twins* if $N[u] = N[v]$. For each vertex v, we partition $N[v]$ into three sets, namely $T(v)$, $D(v)$, and $M(v)$. $T(v)$ consists of v and its true twins. A neighbor u of v is in $D(v)$ if $N[u]$ is a proper subset of $N[v]$. All other neighbors of v are in $M(v)$.

We say that a vertex v is *special* if there is no $u \in M(v)$ such that $D(v) \subseteq N(u)$. Let us partition the set of special vertices of G in such a way that two vertices are in the same part if and only if they are true twins. A set obtained by picking exactly one vertex from each part is called an $S(G)$-*set*. Hence, for any special vertex v, every $S(G)$-set contains exactly one element from $T(v)$. Besides, a graph is called $(G_1, \ldots, G_k)$-free if it contains none of $G_1, \ldots, G_k$ as an induced subgraph.

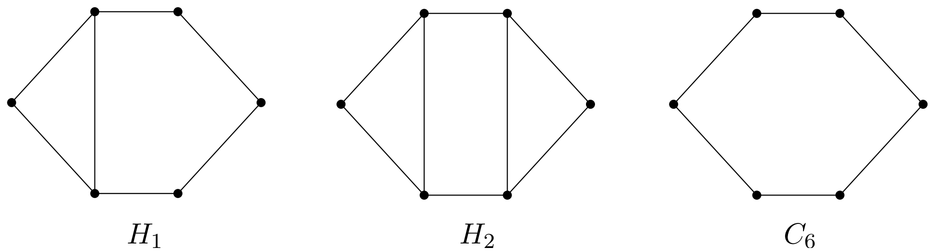

Figure 1: Graphs H_1, H_2, and C_6.

Let H_1 and H_2 be the graphs in Figure 1 and C_6 be a cycle on six vertices. We now state the main theorem of this work:

Theorem 1. *Let G be an (H_1, H_2, C_6)-free graph and S be an $S(G)$-set. Then G is a $(\gamma_t, 2\gamma)$-graph if and only if S is an efficient dominating set of G.*

Remark 1. *Forbidden graphs H_1, H_2, and C_6 are best possible in the sense that if one allows one of these three graphs, then the statement in Theorem 1 is no longer true. For each case, we show counterexamples.*

Note that constructing an $S(G)$-set and checking whether it is an efficient dominating set can be done in polynomial-time. Therefore, the problem of determining whether $\gamma_t(G) = 2\gamma(G)$ for an (H_1, H_2, C_6)-free graph is solvable in polynomial-time.

We have also obtained a characterization of (C_3, C_6)-free graphs with $\gamma_t(G) = 2\gamma(G)$, implying a characterization of C_6-free bipartite graphs with large total domination number. We hence pose the following open question:

Find a characterization of bipartite graphs with $\gamma_t(G) = 2\gamma(G)$.

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CONNECTED COLORING GAME

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A game theoretic variant of graph coloring was introduced independently by S. Brams and H Bodlaender in 1981 and 1991 respectively. Let G be a finite and simple graph. Two players Alice and Bob, in alternate turns are coloring vertices of graph G in a proper way, with Alice playing first, using colors from the given color set C. Alice wins the game when whole graph is colored. Otherwise Bob is the winner. The minimum number of colors for which Alice has a winning strategy is called *game chromatic number* of graph G and denoted as $\chi_g(G)$.

We propose a very new version of the problem. Assume that graph G is connected. We consider similar game on graph G with one extra rule. After each move partial coloring of a graph induced by colored vertices must form a connected graph. In other words in each turn, apart from the first one, players can choose only these vertices which have at least one previously colored neighbor. We call such game as *connected game coloring* of graph G. We also define analogous parameter to the game chromatic number and show bounds for some special families of graphs.

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SOME GRAPH CLASSES WITH TWO-PROPERTY VERTEX ORDERINGS

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There are different methods for defining or characterizing graph classes, including *intersection of properties* and *vertex orderings.* An *intersection of properties* characterization usually takes the form "graph class $\mathcal{G}$ is the intersection of graph classes $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_n$." A *vertex ordering characterization* of a class $\mathcal{G}$ is usually stated in the following way: a graph G is a member of the class $\mathcal{G}$ if and only if G has a vertex ordering simultaneously satisfying properties $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n$.

Both characterization methods allow for the combination of graph properties and when every $\mathcal{G}_i$ has a vertex ordering characterization in terms of property $\mathcal{P}_i$, the two may lead to the same graph class. One example is the class of permutation graphs. A graph G is a permutation graph *iff* it has a single vertex ordering that is comparability and co-comparability [2]. But permutation graphs are also the intersection of the classes of comparability and co-comparability graphs. Interval and split graphs are other known examples of such graph classes [1]. There are cases, however, when the vertex ordering characterization leads to a more restricted graph class. In this talk we will present several such examples, each obtained by combining two properties.

The main example comes from a generalization of the notion of *transitivity.* A *transitively-orientable* (*comparability*) graph is one that has a vertex ordering $(v_1, v_2, \ldots, v_n)$ such that whenever you select 3 vertices v_{i_1}, v_{i_2}, and v_{i_3}, with $i_1 < i_2 < i_3$, the only non-edge cannot be from v_{i_1} to v_{i_3}. In *triangle-extendibility* [3], this definition is changed so that whenever you select 4 vertices $v_{i_1}, v_{i_2}, v_{i_3}$, and v_{i_4}, with $i_1 < i_2 < i_3 < i_4$, you cannot have the only non-edge being from the v_{i_1} to v_{i_4}. There are two natural ways to generalize permutation graphs. One is two take the intersection of triangle-extendible (TE) and co-triangle-extendible (co-TE) graphs (those whose complement is triangle-extendible). Another is to require a single vertex ordering that is a triangle-extendible ordering for a graph G and its complement - we call these graphs doubly-triangle-extendible (DTE). We show that the latter class is a proper subclass of the class TE $\cap$ co-TE

Most of the natural questions are open for both classes, though given the appropriate vertex ordering the clique and independent set problems can be solved in polynomial time. The main open question is recognition of the class. *Counting* - the number of graphs in the class on n vertices is $2^{\Theta(f(n))}$ for which $f(n)$? - is also open. We know that permutation graphs (and more generally, perfect graphs) have either a clique or an independent set of size $cn^{\frac{1}{2}}$. A question related to the counting problem is whether DTE (or TE $\cap$ co-TE) graphs always have a clique or an independent set of size $cn^{\frac{1}{k}}$ for some $k \geq 2$. If the answer to this question is positive, then this would put a non-trivial upper bound on the number of graphs in the class.

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BREAKING GRAPH SYMMETRIES BY EDGE COLOURINGS

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The *distinguishing index* $D'(G)$ of a graph G is the least number of labels in an edge labeling of G that is not preserved by any non-trivial automorphism. We proved in [1] that $D'(G) \leq \Delta(G)$ for every connected graph G, except for C_3, C_4, C_5. Pilśniak [2] characterized all graphs satisfying the equality $D'(G) = \Delta(G)$. In the same paper, she conjectured that

$$D'(G) \leq \left\lceil \sqrt{\Delta(G)} \right\rceil + 1$$

for every 2-connected graph G.

In this talk, we prove this conjecture in a bit stronger form, and present some of its consequences and open problems.

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ON MAXIMAL ROMAN DOMINATION IN GRAPHS

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A Roman dominating function for a graph $G = (V, E)$ is a function $f : V \to \{0, 1, 2\}$ satisfying the condition that every vertex u of G for which $f(u) = 0$ is adjacent to at least one vertex v of G for which $f(v) = 2$. The weight of a Roman dominating function f is the sum $f(V) = \sum_{v \in V} f(v)$, and the minimum weight of a Roman dominating function for G is the Roman domination number, $\gamma_R(G)$, of G. A maximal Roman dominating function for a graph G is a Roman dominating function f such that $V_0 = \{w \in V \mid f(w) = 0\}$ is not a dominating set of G. The maximal Roman domination number, $\gamma_{mR}(G)$, of a graph G equals the minimum weight of a maximal Roman dominating function for G [1].

In this work we give some combinatorial and computational properties concerning the maximal Roman domination number of graphs. For instance, we show that determining the number $\gamma_{mR}(G)$ for an arbitrary graph G is NP-complete even when restricted to bipartite or planar graphs. We describe connected triangle-free graphs G with $\gamma_{mR}(G) = n - 1$ and all trees T of order n such that $\gamma_{mR}(T) = n - 2$. Moreover, we provide a characterization of connected graphs G such that $\gamma_{mR}(G) = \gamma_R(G)$.

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ON (CIRCUIT-)DESTROYABLE GRAPHS

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A *vertex cover* of a simple graph G is a subset of vertices $S \subset V(G)$ which covers all edges, that is, for every pair of adjacent vertices u and v, either u or v belong to S. The minimum size of a vertex cover of G is denoted by $\beta(G)$.

Let G be a connected simple graph. We say that G is *destroyable* or *trail-coverable* if there is a trail P that is vertex cover, if that is the case, we say that P is a *covering trail.* In particular, when the covering trail may be a path, we say *path-destroyable.* We introduce the parameter $\beta_t(G)$ as the minimum size of a vertex cover that induces a trail in G. If G is not destroyable we put $\beta_t(G) = \infty$.

Let G be a connected simple graph with girth $g(G) \geq 3$ and circumference $c(G)$. We say that a G is *circuit-destroyable* or *circuit-coverable* if there is a circuit C in G that is vertex cover of the graph, if that is the case, we say that C is a *covering circuit.* In particular, when the covering circuit may be a cycle, we say *cycle-destroyable.* We also introduce the parameter $\beta_c(G)$ as the minimum size of a vertex cover that induces a circuit in G. If G is not circuit-destroyable we put $\beta_c(G) = \infty$.

In this talk, we will present some minimal graphs, in terms of edges, that are not circuit o trail coverable. We also will study the parameters $\beta_c(G)$ and $\beta_t(G)$ for some families of graphs.

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ON VERTEX-PARITY EDGE-COLORING

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A graph is *odd* if every vertex has an odd degree. Pyber [1] proved that every simple graph can decomposed into at most 4 edge-disjoint odd subgraphs, i.e. that the edges of a simple graph can be colored with 4 colors such that the edges of every color class induce an odd subgraph. Such a coloring is an *odd edge-coloring.* This notion can be generalized in the following way [2]. A vertex signature π of a finite graph G is any mapping $\pi : V(G) \to \{0, 1\}$. An edge-coloring of G is said to be *vertex-parity* for the pair (G, π) if for every vertex v each color used on the edges incident to v appears in parity accordance with π, i.e. an even or odd number of times depending on whether $\pi(v)$ equals 0 or 1, respectively. In the talk, we present a short history of the problem, current results, and applications of the results to other coloring problems.

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INCIDENCE COLORING OF GRAPHS WITH BOUNDED MAXIMUM AVERAGE DEGREE

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An *incidence* of an undirected graph G is a pair (v, e) where v is a vertex of G and e is an edge of G incident to v. Two incidences (v, e) and (u, f) are adjacent if at least one of the following holds: (i) $v = u$, (ii) $e = f$, or (iii) edge vu is from the set $\{e, f\}$. An *incidence coloring* of G is a coloring of its incidences assigning distinct colors to adjacent incidences. The minimum number of colors needed for incidence coloring of a graph is called the *incidence chromatic number*.

Brualdi and Massey [1] conjectured that for every graph G holds $\chi_i(G) \leq \Delta(G) + 2$, but this was disproved by Guiduli [2], who showed that Paley graphs with maximum degree Δ have incidence chromatic number at least $\Delta + \Omega(\log \Delta)$. However, this inequality seems to hold for many graph classes. In this talk we present some results on graphs with prescribed maximum degree and maximum average degree. We show that the incidence chromatic number is at most $\Delta(G)+2$ for any graph G with $\text{mad}(G) < 3$ and $\Delta(G) = 4$, and for any graph with $\text{mad}(G) < \frac{10}{3}$ and $\Delta(G) \geq 8$.

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DECOMPOSITIONS OF COMPLETE MULTIPARTITE GRAPHS INTO MATCHINGS

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A *decomposition* of a graph H is a collection of edge-disjoint subgraphs $G_0, G_1, \ldots, G_t$ of H such that each edge of H belongs to exactly one G_i. We say that H has a G-*decomposition* if each G_i, $i = 1, 2, \ldots, t$, is isomorphic to G. If G and H have the same order and none of vertices is isolated in G then G is a *factor* of H. In particular, if G is regular graph of degree d, it is called a d-*factor* of H. Then a G-decomposition of H is called a d-*factorization*.

A *matching* of *size* n is a set of n independent edges. In particular, a matching of size n in a graph of order $2n$ is a 1-factor (or *perfect* matching), while in a graph of order $2n + 1$ it is a *near 1-factor*. A decomposition into near 1-factors is called a *near 1-factorization*.

Results on the existence of 1-factorizations, near 1-factorizations and in general decompositions into arbitrary matchings of some regular graphs will be presented. Moreover, various algorithmic methods for constructing decompositions, together with their relationship to other combinatorial objects and applications, will be discussed.

ON THE EXISTENCE AND THE NUMBER OF (1,2)-KERNELS IN G-JOIN OF GRAPHS

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Let $k \geq 1$ be an integer. A subset $D \subset V(G)$ is (1,k)-dominating if for every vertex $v \in V(G) \setminus D$ there are $u, w \in D$ such that $uv \in E(G)$ and $d_G(v, w) \leq k$.

A (1,k)-kernel of a graph is a subset of $V(G)$ which is both independent and (1,k)-dominating.

Hedetniemi et al. in [1] gave a sufficient condition for a graph to have a (1,2)-kernel.

Theorem 1. [1] *Every connected graph G having at least two non-adjacent vertices and no triangles has a* (1,2)*-kernel of cardinality* $\alpha(G)$.

In the talk we consider the problem of the existence of (1,2)-kernels in graphs. In particular we give the complete characterization of G-join having a (1,2)-kernel. Moreover we present some results concerning the number of all (1,2)-kernels in G-join.

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TREES WITH EQUAL DOMINATION AND COVERING NUMBERS

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A dominating set of a graph G is a set $D \subseteq V_G$ such that every vertex in $V_G - D$ is adjacent to at least one vertex in D, and the domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. A set $C \subseteq V_G$ is a covering set of G if every edge of G has at least one vertex in C. The covering number $\beta(G)$ of G is the minimum cardinality of a covering set of G. The set of connected graphs G for which $\gamma(G) = \beta(G)$ is denoted by $\mathcal{C}_{\gamma=\beta}$, while $\mathcal{B}$ denotes the set of all connected bipartite graphs in which the domination number is equal to the cardinality of a smaller partite set. A first complete characterization of the set $\mathcal{C}_{\gamma=\beta}$ was given by Hartnell and Rall [2], and independently by Randerath and Volkmann [3]. A simpler characterization was then provided by Wu and Yu [4], and eventually Arumugam et al. [1] proposed another yet characterization, also studying the problem for hypergraphs. In this presentation, we provide alternative characterizations of graphs belonging to the families $\mathcal{C}_{\gamma=\beta}$ and $\mathcal{B}$. Next, we present a constructive characterization of all trees belonging to the set $\mathcal{B}$.

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SEMISTRONG CHROMATIC INDEX

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Besides the notion of a matching of a graph, several other types of more restricted matchings are known. A matching M of a graph G is an *induced matching* if no end-vertices of two edges of M are joined by an edge of G. A proper edge-coloring of a graph in which every color class is an induced matching is called a *strong edge-coloring*. Here, we consider a related edge-coloring with weaker conditions.

A matching M of a graph G is *semistrong* if every edge of M has an end-vertex of degree one in the induced subgraph $G[M]$. A *semistrong edge-coloring* of a graph is a proper edge-coloring in which every color class induces a semistrong matching. This notion was introduced by Gyárfás and Hubenko in [1], who determined the equality between the sizes of maximum induced matchings and maximum semistrong matchings in Kneser and subset graphs. The smallest integer k such that G admits a semistrong edge-coloring with at most k colors is the *semistrong chromatic index* of G.

In this talk, we present currently the best upper bound on the semistrong chromatic index of general graphs, and give tight bounds for trees and graphs with maximum degree 3.

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(DI)GRAPH DECOMPOSITIONS AND LABELINGS: A DUAL RELATION

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A graph G is called edge-magic if there is a bijective function f from the set of vertices and edges to the set $\{1, 2, ..., |V(G)| + |E(G)|\}$ such that the sum $f(x) + f(xy) + f(y)$ for any xy in $E(G)$ is constant. Such a function is called an edge-magic labelling of G and the constant is called the valence. An edge-magic labelling with the extra property that $f(V(G)) = \{1, 2, ..., |V(G)|\}$ is called super edge-magic. A graph is called perfect (super) edge-magic if all theoretical (super) edge-magic valences are possible. In this work, we establish a relationship existing between the (super) edge-magic valences of certain types of bipartite graphs where labelings involving sums are used to characterize the existence of a particular type of decompositions of bipartite graphs. The relation among labelings and decompositions of graphs is not new. In fact, one of the first motivations in order to study graph labelings was the relationship existing between graceful labelings of trees and decompositions of complete graphs into isomorphic trees. What we believe is new and surprising in this work, is the relation between labelings involving sums and graph decompositions. In fact, we believe that this is the first relation found in this direction.

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ELIMINATION PROPERTIES FOR DOMINATING SETS OF GRAPHS

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A set S of vertices of a graph G is a *dominating set* if every vertex of G belongs to S or is adjacent to some vertex of S. A *minimal dominating set* is a dominating set with no proper dominating subsets (see [3]). Since the deletion of a vertex u from a minimal dominating set S results in a non-dominating set of G, we are interested in finding minimal dominating sets S' not containing u close to S in certain sense. This approach can be done in several ways, for example, by considering *exchange* or *elimination* type properties. Roughly speaking, the deletion of a vertex can be seen as a node that fails in a network modeled by the graph, thus in the first case we want to change the vertex that fails by other vertices, whereas in the second case, we do the same but restricting ourselves to vertices belonging to minimal dominating sets containing the vertex that fails. This kind of exchange and elimination properties appear in many other contexts, such as in determining sets and resolving sets of graphs, or in computational geometry, or linear algebra and matroids (see for example [1, 2, 5]).

We study here elimination type properties, concretely, what we call *lower* and *upper* elimination properties. Let $\mathcal{D}(G)$ be the collection of all minimal dominating sets of a graph G and let k be a natural number. We say that G satisfies the *k-lower elimination property* if for every collection of different subsets $D_1, \ldots, D_k \in \mathcal{D}(G)$ and for every $u \in D_1 \cap \ldots \cap D_k$, there is a minimal dominating set included in $(D_1 \cup \ldots \cup D_k) \setminus \{u\}$. Analogously, we say that G satisfies the *k-upper elimination property* if for every $D_1, \ldots, D_k \in \mathcal{D}(G)$ and for every $u \in D_1 \cap \ldots \cap D_k$, there is a minimal dominating set included in $V(G) \setminus \{u\}$ that contains $(D_1 \cup \ldots \cup D_k) \setminus \{u\}$.

We have completely solved this problem for $k = 2$ obtaining, surprisingly, that the same families of graphs satisfy the 2-upper and the 2-lower elimination properties. As a corollary, we characterize all graphs such that their family of minimal dominating sets forms a partition of the vertex set.

This last condition is equivalent to have that every vertex belongs to exactly one minimal dominating set.

Now we present some open problems related to this work. First of all, we are interested in characterizing all graphs satisfying the 1-lower and 1-upper elimination properties. Until now, we have obtained only some partial results. Secondly, another open problem is the study of exchange type properties for minimal dominating sets. Finally, it would be also interesting to study elimination and exchange type properties for other subsets of vertices of a graph, such as independent sets, neighborhoods and cover sets, because of the close relationship between these families and dominating sets using hypergraph operations, such as the transversal and the complementary (see [4]).

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HOMOLOGIES OF DIGRAPHS

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We present a path homology theory for a category of digraphs (graphs) and describe its basic properties. The simplicial homology theory is a particular case of a path homology theory for path complexes. We give basic theorems that are similar to the theorems of algebraic topology and describe relations of our results to the Eilenberg-Steenrod axiomatic. In the talk will be given a number of examples. In particular, the digraph with non-isomorphic cubical singular homology groups and path homology groups will be presented. The main results are published in the papers [1, 2, 3, 4, 5].

An open problem: does there exist a planar graph with a nontrivial 3-dimensional homology group?

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ELECTRIC NETWORKS AND COMPLEX-WEIGHTED GRAPHS

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It is known, that an electric network with resistors can be considered as a weighted graph. In this case the effective resistance of the network can be defined. Due to Kirchoff's law the problem of finding the effective resistance is related to the Laplace operator and the Dirichlet problem on graphs.[1]

In case of alternating current network can contain inductors, capacitors and resistors (passive elements). If one write the inductance and capacity as $i\omega L$ and $\frac{1}{i\omega C}$, keeping writing R for resistance, then the Kirchoff's law holds for the network with passive elements. [2] The problem of finding an effective impedance (analogue of effective resistance) in this case is related with complex-weighted graphs and Dirichlet problem on them.

We introduce the concept of a *complex-weigted graph* and *Laplace operator* on it. Then we introduce the concept of *network* and *Dirichlet problem* related with it. We define an *effective impedance* of the finite network and prove that it satisfies some basic physical properties (Y-Δ transform, parallel law, series law).

The main result is that if the network consists of just passive elements, then its effective impedance has non-negative real part. This result is the corollary of *conservation of complex power*.

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RENDEZVOUS IN A RING WITH A BLACK HOLE, USING TOKENS

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Rendezvous problems ask for a strategy which leads to a meeting of some entities in an environment. From this broad definition stem many models with different properties and numerous results of various palpability. This talk covers a model stronger than proposed in [2], derived from the environment for problem of graph exploration in [1].

The environment is a ring (modeled by a cycle) with a dangerous node, called *black hole*, which destroys agents. While in the original model agents were allowed to store and share informations via memory on nodes, called whiteboards, the model proposed here restricts their size to a single bit, which we model by each agent being equipped with a token which can be dropped or taken from a node. Similar idea was used in the exploration problem [3]. Additionally agents can communicate only when they are present on the same node. Results concerning feasibility are analogous when the number of agents is known, however efficiency of proposed solutions, measured in the amount of moves performed by all agents, are significantly different.

This talk doesn't cover the problem of rendezvous when size of the ring is known instead of the number of agents. A problem which appears to be significantly harder to tackle concerns rendezvous in an arbitrary graph with a black hole.

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SOME RESULTS ON THE PALETTE INDEX OF GRAPHS

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A *proper edge-coloring* of a graph G is a mapping $\alpha : E(G) \rightarrow \mathbf{N}$ such that $\alpha(e) \neq \alpha(e')$ for every pair of adjacent edges $e, e' \in E(G)$. If α is a proper edge-coloring of a graph G and $v \in V(G)$, then the *palette of a vertex* v, denoted by $S(v, \alpha)$, is the set of all colors appearing on edges incident to v. For a proper edge-coloring α of a graph G, we define $S(G, \alpha)$ as follows: $S(G, \alpha) = \{S(v, \alpha) : v \in V(G)\}$. Clearly, for every graph G and its proper edge-coloring α, we have $1 \leq |S(G, \alpha)| \leq |V(G)|$. In [5], Burris and Schelp introduced the concept of vertex-distinguishing proper edge-colorings of graphs. A proper edge-coloring α of a graph G is a vertex-distinguishing proper edge-coloring if for every pair of distinct vertices u and v of G, $S(u, \alpha) \neq S(v, \alpha)$. This means that if α is a vertex-distinguishing proper edge-coloring of G, then $|S(G, \alpha)| = |V(G)|$. On the other hand, Horňák, Kalinowski, Meszka and Woźniak [6] initiated the investigation of the problem of finding proper edge-colorings of graphs with the minimum number of distinct palettes. For a graph G, we define the *palette index* $\check{s}(G)$ of a graph G as follows: $\check{s}(G) = \min_{\alpha} |S(G, \alpha)|$, where minimum is taken over all possible proper edge-colorings of G. In [6], the authors proved that $\check{s}(G) = 1$ if and only if G is regular and $\chi'(G) = \Delta(G)$. From here and the result of Holyer it follows that for a given regular graph G, the problem of determining whether $\check{s}(G) = 1$ or not is NP-complete. Moreover, they also proved that if G is regular, then $\check{s}(G) \neq 2$. In [6], Horňák, Kalinowski, Meszka and Woźniak determined the palette index of complete and cubic graphs. Recently, Bonvicini and Mazzuoccolo [4] studied the palette index of 4-regular graphs. In particular, they constructed 4-regular graphs with palette index 4 and 5. In [2], Bonisoli, Bonvicini and Mazzuoccolo obtained the tight upper bound on the palette index of trees. In [3], Horňák and Hudak determined the palette index of complete bipartite graphs $K_{m,n}$ with $\min\{m, n\} \leq 5$.

Vizing's edge coloring theorem yields an upper bound on the palette index of a general graph G with maximum degree Δ, namely that $\check{s}(G) \leq 2^{\Delta+1} - 2$. However, this is probably far from being tight. Indeed, Avesani et al. [1] described an infinite family of multigraphs whose palette index grows asymptotically as Δ^2; it is an open question whether there are such examples without multiple edges. Furthermore, they suggested to prove that there is a polynomial $p(\Delta)$ such that for any graph with maximum degree Δ, $\check{s}(G) \leq p(\Delta)$. In fact, they suggested that such a polynomial is quadratic in Δ. We thus arrive at the following conjecture: there is a constant C, such that for any graph G with maximum degree Δ, $\check{s}(G) \leq C\Delta^2$.

In this talk we give various upper and lower bounds on the palette index of G in terms of the vertex degrees of G, particularly for the case when G is a bipartite graph with small vertex degrees. Here, we also determine the palette index of grids. Some of our results concern (a, b)-biregular graphs; that is, bipartite graphs where all vertices in one part have degree a and all vertices in the other part have degree b. We conjecture that if G is (a, b)-biregular, then $\check{s}(G) \leq 1+\max\{a, b\}$, and we prove that this conjecture holds for several families of (a, b)-biregular graphs. Additionally, we characterize the graphs whose palette index equals the number of vertices.

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PROPER EDGE COLOURINGS DISTINGUISHING ADJACENT VERTICES – LIST EXTENSION

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Let $G = (V, E)$ be a graph. Consider an edge colouring $c : E \to C$. For a given vertex $v \in V$, by $E(v)$ we denote the set of all edges incident with v in G, while the set of colours associated to these under c is denoted as:

$$S_c(v) = \{c(e) : e \in E(v)\}. \tag{1}$$

The colouring c is called *adjacent vertex distinguishing* if it is proper and $S_c(u) \neq S_c(v)$ for every edge $uv \in E$. It exists if only G contains no isolated edges. The least number of colours in C necessary to provide such a colouring is then denoted by $\chi'_a(G)$ and called the *adjacent vertex distinguishing edge chromatic number* of G. Obviously, $\chi'_a(G) \geq \chi'(G) \geq \Delta$, where Δ is the maximum degree of G, while it was conjectured [3] that $\chi'_a(G) \leq \Delta + 2$ for every connected graph G of order at least three different from the cycle C_5. Hatami [1] proved the postulated upper bound up to an additive constant by showing that $\chi'_a(G) \leq \Delta + 300$ for every graph G with no isolated edges and with maximum degree $\Delta > 10^{20}$.

Suppose now that every edge $e \in E$ is endowed with a list of available colours L_e. The *adjacent vertex distinguishing edge choice number* of a graph G (without isolated edges) is defined as the least k so that for every set of lists of size k associated to the edges of G we are able to choose colours from the respective lists to obtain an adjacent vertex distinguishing edge colouring of G. We denote it by $\mathrm{ch}'_a(G)$. Analogously as above, $\mathrm{ch}'_a(G) \geq \mathrm{ch}'(G)$, while the best (to my knowledge) general result on the classical edge choosability implies that $\mathrm{ch}'(G) = \Delta + O(\Delta^{\frac{1}{2}} \log^4 \Delta)$, see [2]. Extending the thesis of this, a four-stage probabilistic argument granting $\mathrm{ch}'_a(G) = \Delta + O(\Delta^{\frac{1}{2}} \log^4 \Delta)$ for the class of all graphs without isolated edges shall be presented during the talk.

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A CLOSED KNIGHT'S TOUR PROBLEM ON SOME $(M, N, K, 1)$-RECTANGULAR TUBES

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The $m \times n$ *chessboard* is an array with square arranged in m rows and n columns. The standard chessboard is 8×8. The legal move of the knight on the $m \times n$ chessboard is the moving from one square vertically or one square hoirzontally and then two squares move at 90 degrees angle. The problem of the knight's move is "which chessboard that the knight can move from square to square exactly once and return to its starting position?" We call such knight's moves a *closed knight's tour*. The author in [4] answered the question in 1991. A closed knight's tour of a normal two-dimensional chessboard by using legal moves of the knight has been generalized in several ways. One way is to consider a closed knight's tour on a ringboard of width r (see [5]), which is the $m \times n$ chessboard with the middle part missing and the rim contains r rows and r columns. Another way is to stack k copies of the $m \times n$ chessboard to construct an $m \times n \times k$ rectangular chessboard and the closed knight's tour can be on the surface or within the $m \times n \times k$ rectangular chessboard (see [1] and [2]). We combines these two ideas by stacking n copies of $m \times n$ ringboard of width r, which we call the (m, n, k, r)-*rectangular tube* and each stacking copy is called the *level* of (m, n, k, t)-tube. We consider the knight's move within the chessboard, that is, the knight can move in the same level with a legal move, or move one or two squares in the same level and then two or one square in the next consecutive level.

In this talk, we show an algorithm for a closed knight's tour for $(3, 3, k, 1)$-rectangular tube and give a new algorithm for a closed knight's tour for $(4, 4, k, 1)$-rectangular tube which is shorter than [3]. We show the sufficient and necessary conditions for $(3, n, k, 1)$-tube when $n \neq 5$ and $(5, 5, k, 1)$-tube when $k \neq 5$. Moreover, closed knight's tours for $(3, 5, k, 1)$-tube when $k \equiv 0 \pmod 4$ and $(m, m, k, 1)$-tube when $m(> 5)$ is odd and $k \equiv 0 \pmod 4$ are obtained.

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ON THE CONNECTED AND WEAKLY CONVEX DOMINATION NUMBERS OF A GRAPH

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Let $G = (V, E)$ be a connected graph. A subset D of V is *dominating* in G if every vertex of $V - D$ has at least one neighbor in D. A subset D of V is *connected dominating* if D is dominating and the subgraph $G[D]$ induced by D is connected. The minimum cardinality of a connected dominating set of G is a *connected domination number* of G. The *distance* $d_G(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest uv–path in G. A uv–path of length $d_G(u, v)$ is called *uv–geodesic*. A set X is *weakly convex* in G if for any two vertices $a, b \in X$ there exists an ab–geodesic such that all of its vertices belong to X. A set $X \subseteq V$ is a *weakly convex dominating set* if X is weakly convex and dominating. The *weakly convex domination number of a graph* G equals to the minimum cardinality of a weakly convex dominating set in G.

We study the relationship between the weakly convex domination number and the connected domination number of a graph. We focus our attention on the graphs for which the weakly convex domination number equals the connected domination number.

We also analyse the influence of the edge removing on the weakly convex domination number, in particular we show that a weakly convex domination number is an interpolating function.

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GLOBAL EDGE ALLIANCES IN TREES

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In the talk we give a survey of our recent results on the minimum global edge alliances [4] in trees: the formulae to compute the exact size of the minimum global edge alliances in complete k-ary trees, and some upper bounds on the minimum global edge alliances in general trees [5]. We also compare the global edge alliance model with other domination-related problems in trees: total dominating sets [1], alliances [2] and defensive sets [3].

For a given graph G and a subset S of a vertex set of G we define for every subset X of S the predicate $SEC(X) = true$ iff $|N[X] \cap S| \geq |N[X] \setminus S|$ holds, where $N[X]$ is a closed neighbourhood of X in G.

Set S is an edge alliance iff $G[S]$ has no isolated vertices and for each edge $e = \{u, v\} \in E(G[S])$ we have $SEC(\{u, v\}) = true$. Set S is a global edge alliance if it also dominates G.

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ON 2-DOMINATING KERNELS IN GRAPHS AND THEIR PRODUCTS

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A subset $J \subseteq V(G)$ is a 2-dominating kernel of a graph G if J is independent and 2-dominating, i.e. each vertex from $V(G) \setminus J$ has at least two neighbours in J.

In the talk we present classes of graphs which possess 2-dominating kernel and we give results concerning the number and the cardinality of these kernels. We shall show relations between the maximum independent set and the 2-dominating kernel. We also present different, complete characterizations of trees which possess the unique 2-dominating kernel.

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ON ONE-PARAMETER GENERALIZATION OF TELEPHONE NUMBERS

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In the talk we will present one-parameter generalization of classical telephone numbers given by the recurrence relation of the form

$$T_p(n) = T_p(n-1) + p(n-1)T_p(n-2)$$

with initial conditions $T_p(0) = T_p(1) = 1$. We give few graph interpretations of these numbers, their matrix generators and some properties.

GRAPH-BASED MODELLING OF PLANETARY GEARS

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The present paper is dedicated to graph-based models of chosen mechanical objects i.e. planetary gears. In general, graphs can be used for modelling of different engineers tasks and artifacts. There are known graph models of assembly procedure, movements of robot arm equipped in adequate device to perform e.g. welding or machining, layout of machines etc. One can ask: how it could be possible to use a graph as a model of mechanical object? The answer is simple because a graph is equivalent to a relation. In general, mechanical system can be analyzed in an aspect of layout or operation or maintenance schedule etc. The first step in such analysis could be abstraction i.e. discretization aiming for distinguishing the main parts and their mutual connections so in consequence we obtain the set of system discrete parts and the set of pairs of these elements being in particular relationships. Furthermore, we obtain a relation being a simplified model of considered system. It is obvious that during abstraction and discretization some aspects of the system are neglected. Therefore, a particular model is useful for an analysis in a chosen aspect of the object behavior or for a special purpose e.g. kinematical analysis or synthesis. So called bond graphs are the most widely used for modelling versatile mechanical systems. There are known different graphs utilized for modelling of gears: signal flow graphs, contour graphs as well as mixed graphs. The bond graph modeling is well known, the reviews of graph-based modelling of gears can be found in papers [1][2]. Thesis [4] and book [3] contain versatile examples of utilizing of graphs in many engineer problems. The scheme of modelling activities is as follows: (i) abstraction: distinguishing main parts, neglecting some aspects e.g. friction or heating or surface roughness; in case of gears - considering rotating parts i.e. sun wheels, planetary wheels, crown wheels and carriers, (ii) discretization: creation a set of discrete parts; in case of gears neglecting some parameters like complete dimensions of mentioned elements; in case of gears - remaining are numbered items and selected data i.e. module and radiuses; (iii) formulation of relation and subrelations: creation of pairs of co-operating parts and types of co-operation/connections; in case of gears proposed types of connections are: meshing (contact of teeth of toothed parts) e.g. between sun wheel and planetary wheel, turning around the common axes, creation of a pair carrier and planetary wheel,

(iv) turning relationship into graph: drawing a graph by means of some additional rules, in case of gears - parts are vertices, connections are edges, numeration of edges and vertices is performed in accordance with mechanical aspects and properties, drawing a graph consists in usage of special edges line i.e. dashed for meshing, continuous for pair carrier and planetary wheel, double for braked part etc., (v) distinguishing some subgraphs: subgraphs represent some essential aspects of the considered model; in case of graphs the so called f-cycles represent the basic internal gear, path from the input vertex /representing the element connected to the electric motor/ to the output vertex /representing the element connected to the machine/ is the essential route for passing the movement throughout the considered gear; (vi) inter-discipline knowledge transfer phase 1: establishing of connections mechanics graph theory: recognition of meaning of f-cycle as representing an elementary gear and establishing of physical rules of its motions; (vii) inter-discipline knowledge transfer phase 2: creation of algebraic equations, establishing of the system of equations and adding the equations representing working conditions e.g. braking parts adequate rotational velocities are equal to zero; (viii) utilization of graph model: solution of the system of equations and interpretation of results in the mechanical domain e.g.: ratio (+) means compatibility of rotations, sign (-) means reverse drive for the gear. In the presentation, the described rules are utilized for modelling particular planetary gears as well as automobile automatic gear boxes.

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CONSECUTIVE COLOURING OF DIGRAPHS

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A proper edge colouring of a graph with natural numbers is consecutive if colours of edges incident with each vertex form an interval of integers.

In this talk we generalize the previous parameter for digraphs. We present examples of infinite families of oriented graphs that are always consecutively colored and examples of digraphs for which it is not possible to give a consecutive colouring. In addition we give a relation between the consecutive colouring of a digraph and those of its underlying graph.

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