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Monotonous and strong monotonous properties of some propositional proof systems for Classical and Non Classical Logics

For some propositional proof system of classical and non-classical logics we investigate the relations between the lines (t -complexities) and sizes (l -complexities) of proofs for minimal tautologies, which are not a substitution of a shorter tautology of this logic, and results of a substitutions in them. For every minimal tautology φ of fixed logic by $S(\varphi)$ is denoted the set of all tautologies, which are results a substitution in φ .

Definition. The proof system Φ is called t -monotonous (l -monotonous), if for every minimal tautology φ of this system and for every formula ψ from $S(\varphi)$ $t^\Phi(\varphi) \leq t^\Phi(\psi)$ ($l^\Phi(\varphi) \leq l^\Phi(\psi)$).

Definition. The proof system Φ is called t -strong monotonous (l -strong monotonous), if for every non-minimal tautology ψ of this system there is such minimal tautology φ of this system such that ψ belong to $S(\varphi)$ and $t^\Phi(\psi) \leq t^\Phi(\varphi)$ ($l^\Phi(\psi) \leq l^\Phi(\varphi)$).

Formerly it is proved in [1], that Frege systems for classical and non-classical logics are neither t -monotonous nor l -monotonous.

Now we consider the following systems: propositional resolution systems RC , RI , RJ for classical, intuitionistic and Johansson's logics accordingly, eliminations systems $E?$, EI , EJ , based on the determinative normal forms for the same logics [2], and the system GS , based on generalization of splitting method [3].

Theorem. *The systems RC , RI and RJ are t -strong monotonous (l -strong monotonous), but neither of them is t -monotonous (l -monotonous).*

Theorem. *Each of the systems EC , EI , EJ and GS is neither t -monotonous (l -monotonous) nor t -strong monotonous (l -strong monotonous).*

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Bibliography

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