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STRUCTURAL OPTIMIZATION OF AN INHOMOGENEOUS INFINITE LAYER IN PROBLEMS ON PROPAGATION OF PERIODIC WAVES

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A structural optimization problem for an infinite elastic layer inhomogeneous across its finite thickness is considered in order to find the optimum distribution functions for its elastic modulus and density that allows the propagation of a plane periodic wave with a given phase speed. The problem is mathematically formulated as a coupled system of bilinear partial differential equations of the second order with variable controlled coefficients, and the maximum absolute value of the unknown function is taken as the functional to be minimized. The optimum distribution of the elastic modulus across the layer thickness is found as a piecewise constant function. The problem is reduced to a problem of nonlinear programming under constraints of equality type. The results of numerical calculations are presented.

Introduction

At present, along with optimization of constructions, another applied branch of optimization theory is intensely being developed, namely optimization of the structure and topology of constructions [1-3]. In the direct statement, the problem of topological optimization requires that some functional — the optimality criterion, which mostly describes the distribution of material in the volume of a construction, be optimized at given differential (equations of motion, of static equilibrium, etc.) and topological, i.e., geometrical (the spatial domain, not necessarily simply connected, occupied by the construction), restrictions [1, 3]. The principle of solution of the problems on topological optimization consists in reducing them to the

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corresponding, generally nonlinear, programming problems, which can be solved by the effective numerical methods of nonlinear programming [4].

Topology optimization of a construction can lead not only to an advantageous distribution of material in its volume, which is of direct practical importance, preserving or even optimizing its target properties; it also can directly affect the processes in which it is involved [5-7]. For example, by topology optimization of an adhesive layer, deforming in the condition of pure shear, which connects a finite elastic bar to a motionless rigid foundation, the forced vibrations of the bar can be suppressed within a given interval of time without applying additional forces [5]. The problem is mathematically formulated as the problem of optimum control for the Klein–Gordon bilinear equation with controlled coefficient. It is found that, in the sense of minimum intensity of distribution of the adhesive layer, optimum is a piecewise contact between the bar and the rigid foundation. The solution of the problem is reduced to determining the points of switching of the function of optimum control from a system of equality-type nonlinear restrictions and is carried out numerically by the methods of nonlinear programming. The numerical analysis performed revealed that, with increasing shear modulus of the adhesive material, the switching points of the function of optimum control corresponding to the ends of glued sections took very close values, which was in agreement with the discrete contact between the bar and the rigid foundation. By optimizing the structure of a rigid inclusion in an elastic body, it is possible to minimize the sensitivity of the body to the varying shape of the inclusion and to ensure static retardation of a finite crack of any form cropping out at the boundary between the inclusion and body [6]. Optimizing the form of the contact zone between two elastic bodies, it is possible to achieve an almost constant distribution of contact stresses, thus excluding premature separation of the contacting bodies [7].

In [8] is considered a three-dimensional problem on determination of the elastic characteristics of the middle reinforcing layer of a three-layer plate with a symmetric structure in which a harmonic wave propagates with the phase speed equal to that of a wave propagating in the corresponding homogeneous plate without a midlayer. The problem is to find the value of the parameter entering into the dispersion equation and admitting a solution preassigned.

In the present study, the structure of an infinite layer inhomogeneous across its thickness is optimized to determine the optimum distribution function of its elastic characteristics which allows the propagation of a plane harmonic wave with a given phase speed. The control function is given by the intensity of distribution of the elastic modulus of layer material, but the optimum criterion to be minimized — by its maximum value. The problem is to estimate the coefficient of a two-dimensional system of interconnected partial differential equations.

1. In a Cartesian system of coordinates Ox_*z_*, we will consider an elastic infinite layer of constant thickness $2h$, enclosed between stress-free planes $z_* = \pm h,$. Let us assume that the layer is inhomogeneous across the thickness, i.e., its density ρ and elastic modulus E are functions of the z coordinate: $\rho_* = \rho_*(z_*),\ E_* = E_*(z_*),$ but the Poisson ratio ν is constant: $\nu = \mathrm{const}.$ Then, the equations of motion of the layer in the dimensionless variables and functions

$$x = \frac{x_*}{h},\ z = \frac{z_*}{h},\ t = \frac{c_0 t_*}{h},\ w_1 = \frac{w_{1*}}{h},\ \ w_2 = \frac{w_{2*}}{h},\ u = \frac{E_*}{E_0} = \frac{\rho_*}{\rho_0},\ c_0^2 = \frac{E_0}{\rho_0}$$

can be presented in the form

$$\begin{cases} A\dfrac{\partial}{\partial z}\left[u(z)\dfrac{\partial w_1}{\partial z}\right] + \dfrac{\partial}{\partial z}\left[u(z)\dfrac{\partial w_2}{\partial x}\right] + (1+A)u(z)\dfrac{\partial^2 w_1}{\partial x^2} - (1-A)u'(z)\dfrac{\partial w_2}{\partial x} = A(3-A)u(z)\dfrac{\partial^2 w_1}{\partial t^2}, \\[2ex] (1+A)\dfrac{\partial}{\partial z}\left[u(z)\dfrac{\partial w_2}{\partial z}\right] + \dfrac{\partial}{\partial z}\left[u(z)\dfrac{\partial w_1}{\partial x}\right] + A\cdot u(z)\dfrac{\partial^2 w_2}{\partial x^2} - \dfrac{1}{2}(1+A)u'(z)\dfrac{\partial w_1}{\partial x} = A(3-A)u(z)\dfrac{\partial^2 w_2}{\partial t^2}, \end{cases} \quad (1)$$

$$x \in (-\infty, \infty),\ z \in [-1,1],\ t > 0,$$

where $A = 1 - 2\nu$; E_0 and ρ_0 are the reference (calibration) Young's modulus and density.

The boundary conditions on the faces $z = \pm 1$ are

$$\sigma_{33}(x,\pm 1,t) = \left[\frac{u(z)}{A(3-A)} \cdot \left((1-A)\frac{\partial w_1}{\partial x} + (1+A)\frac{\partial w_2}{\partial z}\right)\right]\Bigg|_{z=\pm 1} = 0,$$

$$\sigma_{13}(x,\pm 1,t) = \left[\frac{u(z)}{3-A} \cdot \left(\frac{\partial w_1}{\partial z} + \frac{\partial w_2}{\partial x}\right)\right]\Bigg|_{z=\pm 1} = 0, \tag{2}$$

$$x \in (-\infty,\infty),\ t > 0.$$

In Eqs. (1) and (2), $w_1 = w_1(x,z,t)$ and $w_2 = w_2(x,z,t)$ are the horizontal and vertical displacements of layer points; $\sigma_{33}(x,z,t)$ and $\sigma_{13}(x,z,t)$ are the normal and tangential stresses.

The basic purpose of this study is to determine a function $u^{\circ}(z)$, from a set U of allowable functions u, which, for the bilinear system (1), ensures a harmonic solution with a given phase speed c, equal to that of the wave propagating in a homogeneous layer with the elastic modulus E_0 and density ρ_0, which minimizes the optimality criterion [5, 9]

$$\kappa[u] = \sup_{z\in[-1,1]} u(z),\quad u \in U. \tag{3}$$

We should note that the control function describing the distribution intensity of elastic modulus and density of the layer across its thickness enters both into the bilinear system (1), with its first power and first-order derivative, and into boundary conditions (2).

Based on physical considerations, we assume that the required function is strictly positive. It is also easily seen that the function $u(z)$ is finite with the carrier $[-1, 1]$, i.e., it identically becomes zero outside this closed interval. Further, since the space of summable functions $L^{\infty}[-1, 1]$ is the Banach space relative to norm (3), it is expedient to consider a set of finite on $[-1, 1]$, essentially bounded, positive functions $U \subset L^{\infty}[-1,1]$ as the set of admissible controls U.

2. Presenting the solution to system (1) in the form [10, 11]

$$w_1(x,z;t) = \frac{\partial \Phi^*}{\partial x} - \frac{\partial \psi^*}{\partial z},\ w_2(x,z;t) = \frac{\partial \Phi^*}{\partial z} + \frac{\partial \psi^*}{\partial x},$$

$$x \in (-\infty,\infty),\ z \in [-1,1],\ t > 0,$$

$$\Phi^*(x,z;t) = \left[a_1\cosh(\sigma z) + b_1\sinh(\sigma z)\right]e^{ik(x-c_1 t)} \equiv \Phi(z)e^{ik(x-c_1 t)},$$

$$\psi^*(x,z;t) = \left[a_2\cosh(\varsigma z) + b_2\sinh(\varsigma z)\right]e^{ik(x-c_1 t)} \equiv \psi(z)e^{ik(x-c_1 t)},$$

$$\sigma^2 = k^2\left(1-c_{\sigma}^2\right),\ \varsigma^2 = k^2\left(1-c_{\varsigma}^2\right),\ c_{\sigma} = \frac{c}{c_l},\ c_{\varsigma} = \frac{c}{c_t},\ c_1 = \frac{c}{c_0}$$

and inserting it into Eq. (1), after simple algebraic transformations, we have

$$\begin{cases} u'(z)\Gamma_{11}(z;k) + u(z)\Gamma_{12}(z;k) = u(z)\Gamma_{13}(z;k,c_1), \\ u'(z)\Gamma_{21}(z;k) + u(z)\Gamma_{22}(z;k) = u(z)\Gamma_{23}(z;k,c_1), \end{cases} z \in [-1,1]. \tag{4}$$

where c_l and c_t are the propagation speeds of longitudinal and transverse waves in the layer, $k = hk_*$, k_* is the wavenumber, and a_j and b_j $(j = 1;2)$ are found from conditions (2):

$$a_1 = a_2 \frac{a_{22}}{a_{21}} \frac{\cosh \varsigma}{\cosh \sigma}, \quad b_1 = b_2 \frac{a_{22}}{a_{21}} \frac{\sinh \varsigma}{\cosh \sigma}, \quad a_2 = -b_2 \frac{a_{21} a_{12}}{a_{11} a_{22}} \tanh \sigma,$$

$$a_{11} = (1+A)\sigma^2 + (1-A)k^2, \; a_{12} = 2i\varsigma Ak, \; a_{21} = 2i\sigma k, \; a_{22} = \varsigma^2 + k^2.$$

The phase speed c has to satisfy the dispersion equation $a_{11}^2 a_{22}^2 - a_{12}^2 a_{21}^2 = 0$ of the problem. Earlier, the following designations were assumed:

$$\Gamma_{11}(z;k) = A\left[2ik\frac{d\Phi}{dz} - \left(\frac{d^2}{dz^2} + k^2\right)\psi\right],$$

$$\Gamma_{13}(z;k,c_1) = A(3-A)kc_1\left[kc_1 \cdot \Phi + i\frac{d\psi}{dz}\right],$$

$$\Gamma_{12}(z;k) = ik(1+A)\left[\frac{d^2}{dz^2} - k^2\right]\Phi - A\left[\frac{d^3}{dz^3} - k^2\frac{d}{dz}\right]\psi,$$

$$\Gamma_{21}(z;k) = \left[(1+A)\frac{d^2}{dz^2} - \frac{1}{2}(1-A)k^2\right]\Phi + i\frac{1}{2}(3-A)k\frac{d\psi}{dz},$$

$$\Gamma_{22}(z;k) = (1+A)\left[\frac{d^3}{dz^3} - k^2\frac{d}{dz}\right]\Phi + iAk\left[\frac{d^2}{dz^2} - k^2\right]\psi,$$

$$\Gamma_{23}(z;k,c_1) = A(3-A)kc_1\left[i\frac{d\Phi}{dz} + kc_1 \cdot \psi\right].$$

It is obvious that the system of differential equations (4) has no solutions in the class of ordinary functions. Since the coefficients of the sought-for function and of its derivative are concentrated on $[-1,1]$, the integrals of both parts of Eqs. (4) generate generalized functions concentrated on $[-1,1]$ and equal almost everywhere on this segment. Therefore, after integration by parts, we have

$$\int_{-1}^{1} \Lambda_1(z;k,c_1)u(z)dz = M_1(k), \quad \int_{-1}^{1} \Lambda_2(z,k,c_1)u(z)dz = M_2(k),$$

$$\Lambda_p(z;k,c_1) = \Gamma'_{p1}(z;k) - \Gamma_{p2}(z;k) + \Gamma_{p3}(z;k,c_1), \tag{5}$$

$$M_p(k) = \left[u(z)\cdot\Gamma_{p1}(z;k)\right]\Big|_{-1}^{1}, \; p = 1, 2.$$

Separating now the real and imaginary parts of Eqs. (5), we come to the following system of restrictions on the function required:

$$\int_{-1}^{1} \Lambda_1^{\mathrm{Re}}(z;k,c_1)u(z)dz = M_1^{\mathrm{Re}}, \quad \int_{-1}^{1} \Lambda_2^{\mathrm{Re}}(z;k,c_1)u(z)dz = M_2^{\mathrm{Re}},$$

$$\int_{-1}^{1} \Lambda_1^{\mathrm{Im}}(z;k,c_1)u(z)dz = M_1^{\mathrm{Im}}, \quad \int_{-1}^{1} \Lambda_2^{\mathrm{Im}}(z;k,c_1)u(z)dz = M_2^{\mathrm{Im}}, \tag{6}$$

where the superscript $\mathrm{Re}\,(\mathrm{Im})$ designates the real (imaginary) part of a quantity.

Thus, solution of the problem examined has been reduced to determination of a positive function $u^{o}(z)$, optimum in sense (3), which satisfies the system of integral restrictions (6). Since the functions $\Lambda_1^{\mathrm{Re}}(z;k,c_1)$, $\Lambda_2^{\mathrm{Re}}(z;k,c_1)$, $\Lambda_1^{\mathrm{Im}}(z;k,c_1)$, and $\Lambda_2^{\mathrm{Im}}(z;k,c_1)$, are obviously summable at $z\in[-1,1]$, the solution of the resulting problem can be constructed, for example, by the method described in [9], treating these restrictions as a problem of moments in the space of functions $L^1[-1,1]$, which is conjugate with the space of control functions. Employing the general theory of moments, it is possible to show that the sought-for function, optimum in the sense of criteria (3), is a piecewise-constant one with discontinuities at switching points — the points where its value jumps from one level to another [5, 9, 12]. Let us designate these switching points as $-1=z_1^{o}<z_2^{o}<\ldots<z_n^{o}<z_{n+1}^{o}=1$. Then, the required function, contrary to that given in [5], is written as

$$u^{o}(z)=\sum_{j=1}^{n}u_j^{o}\left[H\left(z-z_j^{o}\right)-H\left(z-z_{j+1}^{o}\right)\right],\quad u_j^{o}>0,\ \left\{z_j^{o}\right\}_{j=2}^{n}\subset(-1,1), \tag{7}$$

where

$$H(z)=\begin{cases}1,\ z>0,\\ 0,\ z<0\end{cases}$$

is the Heaviside step function. Insertion of (7) into system (6) yields

$$\sum_{j=1}^{n}u_j^{o}\int_{z_j^{o}}^{z_{j+1}^{o}}\Lambda_1^{\mathrm{Re}}(z;k,c_1)\,dz=M_1^{\mathrm{Re}},\quad \sum_{j=1}^{n}u_j^{o}\int_{z_j^{o}}^{z_{j+1}^{o}}\Lambda_2^{\mathrm{Re}}(z;k,c_1)\,dz=M_2^{\mathrm{Re}},$$

$$\sum_{j=1}^{n}u_j^{o}\int_{z_j^{o}}^{z_{j+1}^{o}}\Lambda_1^{\mathrm{Im}}(z;k,c_1)\,dz=M_1^{\mathrm{Im}},\quad \sum_{j=1}^{n}u_j^{o}\int_{z_j^{o}}^{z_{j+1}^{o}}\Lambda_2^{\mathrm{Im}}(z;k,c_1)\,dz=M_2^{\mathrm{Im}}.$$

In view of the designations accepted, this system can be transformed to the form

$$\begin{gathered}u_1^{o}\cdot\Pi_{11}^{\mathrm{Re}}+\sum_{j=2}^{n-1}u_j^{o}\Lambda_{1j}^{\mathrm{Re}}+u_n^{o}\cdot\Pi_{1n}^{\mathrm{Re}}=0,\ u_1^{o}\cdot\Pi_{21}^{\mathrm{Re}}+\sum_{j=2}^{n-1}u_j^{o}\Lambda_{2j}^{\mathrm{Re}}+u_n^{o}\cdot\Pi_{2n}^{\mathrm{Re}}=0,\\ u_1^{o}\cdot\Pi_{11}^{\mathrm{Im}}+\sum_{j=2}^{n-1}u_j^{o}\Lambda_{1j}^{\mathrm{Im}}+u_n^{o}\cdot\Pi_{1n}^{\mathrm{Im}}=0,\ u_1^{o}\cdot\Pi_{21}^{\mathrm{Im}}+\sum_{j=2}^{n-1}u_j^{o}\Lambda_{2j}^{\mathrm{Im}}+u_n^{o}\cdot\Pi_{2n}^{\mathrm{Im}}=0,\end{gathered} \tag{8}$$

where

$$u_1^{o}=u^{o}(-1),\quad u_n^{o}=u^{o}(1),$$

$$\left.\begin{aligned}\Pi_{p1}^{\mathrm{Re}}&=\Lambda_{p1}^{\mathrm{Re}}+\Gamma_{p1}^{\mathrm{Re}}(-1;k),\ \Pi_{pn}^{\mathrm{Re}}=\Lambda_{pn}^{\mathrm{Re}}-\Gamma_{p1}^{\mathrm{Re}}(1;k),\\ \Pi_{p1}^{\mathrm{Im}}&=\Lambda_{p1}^{\mathrm{Im}}+\Gamma_{p1}^{\mathrm{Im}}(-1;k),\ \Pi_{pn}^{\mathrm{Im}}=\Lambda_{pn}^{\mathrm{Im}}-\Gamma_{p1}^{\mathrm{Im}}(1;k),\end{aligned}\right.\quad p=1,2,$$

$$\Lambda_{pj}^{\mathrm{Re}}=\int_{z_j^{o}}^{z_{j+1}^{o}}\Lambda_p^{\mathrm{Re}}(z;k,c_1)\,dz,\ \Lambda_{pj}^{\mathrm{Im}}=\int_{z_j^{o}}^{z_{j+1}^{o}}\Lambda_p^{\mathrm{Im}}(z;k,c_1)\,dz,\quad j=\overline{1;n}.$$

Thus, the problem of structural optimization considered is reduced to the variational problem on calculation of the minimum of functional (3) on the set of the allowable solutions u_j^{o} and z_j^{o}, $j=\overline{1;n}$, of system (8); in other words, we have to determine the positive numbers u_j^{o} forming the minimum set and the points $z_j^{o}\in[-1,1]$ satisfying a system of nonlinear restrictions of type (8). The solution to the problem described can be constructed by the effective approximate methods of nonlinear programming [4].

TABLE 1. Optimum Structure of the Layer

k	u_j^{o}	z_j^{o}
	$c_\varsigma = 0.75$	
0.1	$u_1^{o} = 0.18,\ u_2^{o} = 1.31,\ u_3^{o} = 1.43,\ u_4^{o} = 0.2$	$z_2^{o} = -0.6,\ z_3^{o} = -0.5,\ z_4^{o} = 0.34$
0.5	$u_1^{o} = 0.4,\ u_2^{o} = 1,\ u_3^{o} = 0.43$	$z_2^{o} = -0.35,\ z_3^{o} = -0.2$
1.0	$u_1^{o} = 0.2,\ u_2^{o} = 0.42,\ u_3^{o} = 0.38$	$z_2^{o} = 0,\ z_3^{o} = 0.79$
π	$u_1^{o} = 0.36,\ u_2^{o} = 0.1,\ u_3^{o} = 0.7,\ u_4^{o} = 0.5$	$z_2^{o} = -0.6,\ z_3^{o} = 0,\ z_4^{o} = 0.91$
2π	$u_1^{o} = 0.7,\ u_2^{o} = 0.45,\ u_3^{o} = 0.9,\ u_4^{o} = 0.27$	$z_2^{o} = -0.8,\ z_3^{o} = 0,\ z_4^{o} = 0.56$
3π	$u_1^{o} = 0.4,\ u_2^{o} = 0.31,\ u_3^{o} = 0.75$	$z_2^{o} = -0.43,\ z_3^{o} = 0.76$
	$c_\varsigma = 0.87$	
0.1	$u_1^{o} = 0.32,\ u_2^{o} = 0.47,\ u_3^{o} = 0.32$	$z_2^{o} = -0.38,\ z_3^{o} = 0.47$
1.0	$u_1^{o} = 0.72,\ u_2^{o} = 0.98,\ u_3^{o} = 0.58,\ u_4^{o} = 0.84$	$z_2^{o} = -0.3,\ z_3^{o} = 0.16,\ z_4^{o} = 0.77$
π	$u_1^{o} = 0.83,\ u_2^{o} = 1.3,\ u_3^{o} = 1.67,\ u_4^{o} = 0.91$	$z_2^{o} = -0.34,\ z_3^{o} = 0.53,\ z_4^{o} = 0.6$
2π	$u_1^{0} = 0.91,\ u_2^{o} = 1.1,\ u_3^{o} = 1.5$	$z_2^{o} = -0.82,\ z_3^{o} = 0.56$
3π	$u_1^{o} = 1,\ u_2^{o} = 2,\ u_3^{o} = 1.62,\ u_4^{o} = 1.37$	$z_2^{o} = -0.3,\ z_3^{o} = 0,\ z_4^{o} = 0.75$
	$c_\varsigma = 0.95$	
0.1	$u_1^{o} = 0.27,\ u_2^{o} = 0.5\ u_3^{o} = 0.23$	$z_2^{o} = -0.04,\ z_3^{o} = 0.85$
0.5	$u_1^{o} = 0.72,\ u_2^{o} = 0.92,\ u_3^{o} = 0.64,\ u_4^{o} = 1$	$z_2^{o} = -0.32,\ z_3^{o} = 0.68,\ z_4^{o} = 0.97$
2π	$u_1^{o} = 4.87,\ u_2^{o} = 1.2,\ u_3^{o} = 0.81,$ $u_4^{o} = 0.67,\ u_5^{o} = 1.2,\ u_6^{o} = 0.12$	$z_2^{o} = -0.92,\ z_3^{o} = -0.08,\ z_4^{o} = 0.131,$ $z_5^{o} = 0.64,\ z_6^{o} = 0.853$
3π	$u_1^{o} = 0.75,\ u_2^{o} = 1,\ u_3^{o} = 1.3,$ $u_4^{o} = 0.9,\ u_5^{o} = 1.5,\ u_6^{o} = 0.36$	$z_2^{o} = -0.81, z_3^{o} = 0,\ z_4^{o} = 0.13,$ $z_5^{o} = 0.62,\ z_6^{o} = 0.85$

3. Let us consider now the procedure for determining the optimum structure of a layer by a particular example with fixed $A = 0.5$ ($\nu = 0.25$) and different values of k. Then, $c_\varsigma^2 = 3c_\sigma^2 = 2.5c_1^2$. As an arbitrary plane problem at $b_2 = 1$, and $c_\varsigma < 1$, we have

$$\Lambda_1^{\mathrm{Re}}(z;k,c_1) = \frac{k}{2}\left[\frac{d^2}{dz^2} - 3k\right]\Phi_{\mathrm{Re}} - k^2\frac{d\psi}{dz},$$

$$\Lambda_1^{\mathrm{Im}}(z;k,c_1) = -\frac{5kc_1}{4}\left[kc_1\Phi_{\mathrm{Re}} + \frac{d\psi}{dz}\right],$$

$$\Lambda_2^{\mathrm{Re}}(z;k,c_1) = \frac{5kc_1}{4}\left[-\frac{d\Phi_{\mathrm{Re}}}{dz} + kc_1\psi\right],$$

$$\Lambda_2^{\mathrm{Im}}\left(z;k,c_1\right)=\frac{5k^2}{4}\frac{d\Phi_{\mathrm{Re}}}{dz}+\frac{k}{2}\left[\frac{3}{2}\frac{d^2}{dz^2}+k^2\right]\psi,$$

$$\Phi_{\mathrm{Re}}\left(z;k\right)=\frac{\varsigma k}{\cosh^2\sigma}\cdot\frac{\cosh\sigma\cosh\varsigma\cosh\left(\sigma z\right)+\sinh\sigma\sinh\varsigma\sinh\left(\sigma z\right)}{3\sigma^2+k^2},$$

$$\psi\left(z;k\right)=\frac{2\sigma\varsigma k^2\tanh\sigma}{\left(3\sigma^2+k^2\right)\left(\varsigma^2+k^2\right)}\cdot\cosh\left(\varsigma z\right)+\sinh\left(\varsigma z\right).$$

Upon insertion of the results obtained into system (8) and addition of equality-type restrictions on the switching points $-1=z_1^{\mathrm{o}}<z_2^{\mathrm{o}}<\ldots<z_n^{\mathrm{o}}<z_{n+1}^{\mathrm{o}}=1$, the numerical experiment has been realized; the results obtained are presented in Table 1.

It is worth mentioning that, for example, at $c_\varsigma=0.75,\ k=0.5$ and $c_\varsigma=0.87,\ k=0.1$, the structure of the layer coincides with that of the reinforced plate examined in [11], but without symmetry.

Conclusions

Consideration of the structural optimization of inhomogeneous media as a problem of optimum control of a coefficient leads to the conclusion that the distribution intensity of the elastic modulus of a layer inhomogeneous across its thickness, which is optimum in the sense of functional (3), can be assigned in the form of a piecewise constant function (7) that is uniquely determined by the solution of a problem of nonlinear programming in the presence of restrictions of type (8). The solution obtained corresponds to a piecewise inhomogeneous medium with a layered structure, and the quantity u_j^{o} characterizes the elastic modulus of a *j*th component of the layer of relative thickness $h_j^{\mathrm{o}}=z_j^{\mathrm{o}}-z_{j-1}^{\mathrm{o}}\ \left(j=\overline{1;n+1}\right)$. A numerical experiment has been performed for different values of the parameter k, describing the wavenumber, and c_ς, describing the ratio between the phase speed of a propagating wave and the propagation speed of transverse waves in the layer.

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