group is minimal 3-generated, i.e. if $H \leq G$ and $H = \langle H \cap D \rangle$ then either $H = G$ or H can be generated by two elements of D. The classification of minimal 3-generated groups will be presented on the talk.

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On pairs of wreath products of groups generating equal varieties

The aim of this communication is to present our recent note [5] which suggests a method of detection of wreath products which generate the same variety of groups. More precisily: we are given the groups A_1, A_2, B_1, B_2 such that A_1 and A_2 generate the same variety $\mathfrak{U}$, and the groups B_1 and B_2 generate the same variety $\mathfrak{V}$. Do the wreath products $A_1 \mathrm{Wr} B_1$ and $A_2 \mathrm{Wr} B_2$ generate the same variety? In other words, does the equality

$$\mathrm{var}(A_1 \mathrm{Wr} B_1) = \mathrm{var}(A_2 \mathrm{Wr} B_2) \qquad (*)$$

hold for the given A_1, A_2, B_1, B_2 (see background information on varieties of groups in [7])? This topic continues our research of [3, 4, 6] in which we discussed other properties of varieties generated by wreath products, in particular, equality of the variety generated by $A_1 \mathrm{Wr} B_1$ to the product $\mathfrak{U}\mathfrak{V}$.

To give an answer to the above question for some classes of nilpotent and abelian groups we need some auxiliary notations for abelian groups (we adopt them from [2]). Let B_1 and B_2 be abelian groups of finite exponent, and let for a prime p their p-primary components $B_1(p)$ and $B_2(p)$ have direct decompositions $B_1(p) = C_{p^{u_1}}^{\mathfrak{m}_{p^{u_1}}} \times \cdots \times C_{p^{u_r}}^{\mathfrak{m}_{p^{u_r}}}$ and $B_2(p) = C_{p^{v_1}}^{\mathfrak{m}_{p^{v_1}}} \times \cdots \times C_{p^{v_s}}^{\mathfrak{m}_{p^{v_s}}}$, where $C_{p^{u_i}}^{\mathfrak{m}_{p^{u_i}}}$ is the direct product of $\mathfrak{m}_{p^{u_1}}$ copies of the cycle $C_{p^{u_i}}$ of order p^{u_i}; and $C_{p^{v_i}}^{\mathfrak{m}_{p^{v_i}}}$ is defined similarly for B_2. Suppose $u_1 > \cdots > u_r$ and $v_1 > \cdots > v_r$. Define $B_1(p) \equiv B_2(p)$ if and only if: either $B_1(p), B_2(p)$ both are *finite* and isomorphic; or $B_1(p), B_2(p)$ both are *infinite*, and there is a k such that: (i) $C_{p^{u_k}}^{\mathfrak{m}_{p^{u_k}}}$ is the first infinite factor of $B_1(p)$, $C_{p^{v_k}}^{\mathfrak{m}_{p^{v_k}}}$ is the first infinite factor of $B_2(p)$; (ii) $u_k = v_k$; (iii) $u_i = v_i$, $\mathfrak{m}_{p^{u_i}} = \mathfrak{m}_{p^{v_i}}$ for each $i = 1, \ldots, k-1$.

Theorem. (Theorem 2.3 in [5]) Let A_1, A_2 be non-trivial nilpotent groups of exponent m generating the same variety, and let B_1, B_2 be non-trivial abelian groups of exponent n generating the same variety. If any prime divisor p of n also divides m, then the equality $(*)$ holds for A_1, A_2, B_1, B_2 if and only if $B_1(p) \equiv B_2(p)$ for each of such primes p.

We have especially simple situations in cases, when B_1, B_2 both are finite, or if one of them is finite, and the other is infinite:

Corollary. In the above notations:

1. equality $(*)$ holds for finite groups B_1, B_2 if and only if B_1 and B_2 are isomorphic,
2. equality $(*)$ never holds if one of the groups B_1, B_2 is finite, and the other is infinite.

The presented facts generalize some results of Kovács and Newman on subvariety structure of product varieties of groups [1].

The proofs technique uses both standard methods of varieties of groups reflected in [7], and also newer methods, such as, Shield's theory developed for computation of the nilpotency class for wreath products [8, 9].

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The ordered projective geometry over skew fields

Let $F =< F, +, \cdot >$ be a skew field. Recall that the skew field F is said to be *a partially ordered skew field* whenever $< F, +, \leqslant>$ is a partially ordered group and the following condition holds:

if $a \leqslant b$ and $c > 0$, then $ac \leqslant bc$ and $ca \leqslant cb$ for all $a, b, c \in F$.

We will call a partially ordered skew field F *a directed skew field* whenever the group $< F, +, \leqslant>$ is a directed group.

Everywhere below, F always denotes a partially ordered skew field, the characteristic of F is equal to zero, and the unity element of the field F is a positive element.

One can find the next definition in books [1] and [2].

A left vector space $_FV =< V, +, \{\alpha |\ \alpha \in F\} >$ over a partially ordered skew field F is said to be *a partially ordered vector space* whenever the group $< V, +, \leqslant>$ is a partially ordered group and the following condition holds:

if $0 \leqslant v$, then $0 \leqslant \alpha v$ for all elements $v \in V$ and all $\alpha > 0$ from the skew field F.

A partially ordered vector space $_FV$ over a partially ordered skew field F is said to be *directed (linearly ordered)* if the group $< V, +, \leqslant>$ is a directed (linearly ordered) group. The vector space $_FV$ is called *a vector lattice* if the group $< V, +, \leqslant>$ is a lattice-ordered group.

The idea of vector lattices was first considered by L. V. Kantorovich [3] and Riesz [5]. The concept of vector lattices is important in the functional analysis. For this reason, properties of real function spaces were investigated by different authors (see [4]).

At present, we have seen a beginning of systematic investigation for partially ordered vector spaces over different partially ordered skew fields.

Let M be a linear subspace of a partially ordered vector space $_FV$ over a partially ordered skew field F. We will call M *a directed subspace* if the group $< M, +, \leqslant>$ is a directed subgroup of the group $< V, +, \leqslant>$.

Let us denote by $L = L(_FV)$ the set of all directed subspaces of a partially ordered vector space $_FV$ over a partially ordered skew field field F.

Theorem 1. Suppose $_FV$ is a partially ordered vector space over a directed skew field F, and $0 < v \in V$; then there exists a directed subspace M_v of the space $_FV$, where any positive element $u \in M_v$ satisfies inequalities $u \leqslant \alpha v$ for some elements $\alpha > 0$ from the skew field F.

A partially ordered group G is referred to *interpolation groups* whenever for inequalities $u_1, u_2 \leqslant v_1, v_2$ there exists an element $w \in G$ for which $u_1, u_2 \leqslant w \leqslant v_1, v_2$ for all $u_1, u_2, v_1, v_2 \in G$.

We will call a partially ordered vector space $_FV$ over a partially ordered skew field F *an interpolation space* if the group $< V, +, \leqslant>$ is an interpolation group.

Positive elements a and b are called to be *almost orthogonal* in a partially ordered group G whenever inequalities $g \leqslant a, b$ imply the validity of inequalities $g^n \leqslant a, b$ for all elements $g \in G$ and