

On harmonic conjugation problem in spaces of quaternion-valued functions

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The classical M. Riesz theorem on harmonic conjugates in the Hardy spaces over the unit disc \mathbb{D} asserts that for a harmonic function u in the Hardy space $h^p(\mathbb{D})$ for some $p, 1 < p < \infty$, its harmonic conjugate v is also in $h^p(\mathbb{D})$, and so the holomorphic function $f = u + iv$ is in $H^p(\mathbb{D})$. Hardy and Littlewood revealed the same harmonic conjugation property for Bergman spaces in \mathbb{D} . Later, analogous results were obtained for various function spaces such as weighted Bergman, Bloch, Dirichlet and others. The problem of harmonic conjugates in the framework of quaternion analysis was studied by Sudbery [1] who established an explicit formula for harmonic conjugates in \mathbb{R}^4 such that a quaternion-valued monogenic function is defined. By another integral formula we construct harmonic conjugates in reduced quaternions in \mathbb{R}^3 and prove the preservation of weighted Bergman and Dirichlet spaces under harmonic conjugation operator over the unit ball in \mathbb{R}^3 . Some preceding results can be found in [2], [3].

References

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