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BURGERS VORTEX IN A PROTOPLANETARY DISK

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The effect of a Burgers vortex on the formation of planetesimals in a protoplanetary disk is examined in a local approximation. It is shown that a Burgers vortex with a uniformly rotating core and a converging radial flow of matter can efficiently accumulate dusty matter with a mass on the order of 10^{27}-10^{28} g in its core region over a characteristic time of ~10^6-10^7 years. In the localization region of the Burgers vortex, the thickness of the disk increases.

Keywords: *protoplanetary disk: Burgers vortex: planetesimals*

1. Introduction

Studies of the radial dependence of the infrared, submillimeter, and centimeter emission of protoplanetary disks show that vortices serve as incubators for the growth of dust particles and the formation of planetesimals of kilometer size [1]. Their nucleation from micro-sized dust probably involves more than one physical process [2]. It is generally believed that particle growth in dusty, circumstellar disks is a hierarchical process [3,4]. The initial stage of growth probably extends from the formation of a dust grain kernel of submicron size from a primordial nebula through the formation of cumulative clusters of dust up to sizes of ~10 cm over a characteristic time on the order of 10^3 years. Further growth by this process is brought to a halt by collisional breakup processes [5,6]. Under these

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conditions the particle dynamics and coagulation are controlled by van der Waals forces and Brownian motion. The best astrophysical evidence of the growth of dust particles to these sizes is the detection of emission at 3.5 cm from dust in a disk with a radius of 225 a.u. facing toward us and located around the classical T Tau star TW Hya (age ~5-10 My) at a distance of 56 pc from us.

In the standard models for protoplanetary disks, the gas pressure decreases along the radius. The gas in the disk essentially moves at Kepler velocities. Solid particles lose angular momentum and energy because of friction in the gas. Particles of meter sizes drift toward the star for a few hundred years, much less than the lifetime of the disk, which is a few million years [8,9].

Long-lived vortex structures in gases are a possible way of concentrating particles with sizes up to ~10 cm and growing them into planetesimals. Similar effects of vortices have been observed on the earth in special laboratories and in the ocean. For example, observations show that oceanic vortices lure fish larvae into a trap not far from the shores of South Australia [10].

In some regions of stratified protoplanetary disks, the flow has a 2D turbulent character. The attractive feature of this kind of hydrodynamic flow is that long-lived vortices develop among the background of fine eddies in it without requiring special initial conditions [11-13]. The formation of Burgers vortices, which are the subject of this paper, are often observed in laboratory experiments on 2D turbulent flows [14]. If protoplanetary disks are capable of developing 2D turbulent flows, then they can form long-lived large-scale vortices with a lifetime on the order of a hundred orbital periods. Anticyclonic vortices in a protoplanetary disk will merge with and reinforce one another, while cyclonic vortices will be destroyed by shear flow.

In an anticyclone, solid particles will be captured by the Coriolis force directed toward the center of the vortex. If the vortex lasts ~100 orbits in the nebula with the mass of the sun, the number of captured particles can approach the mass of a planet (several times the earth's mass). Long-lived vortices in a protoplanetary disk drifting from the outer regions of the disk will enable accumulation of the mass necessary for formation of the core of a giant planet [15,16].

This paper examines Burgers vortices in protoplanetary disks and their role in the formation of planetesimals.

In a cylindrical coordinate system (r, θ, z) a Burgers vortex is defined by

$$v_r = -Ar, \quad v_\theta = \omega r_0^2 \left[1 - \exp\left(-r^2/r_0^2\right)\right]/r, \quad v_z = 2Az \tag{1}$$

and consists of a vortex with a flow of matter converging toward its center, where A characterizes the converging flow, and ω and r_0 are the circulation and size of the vortex trunk. The rotation in the reign of the trunk of the vortex is essentially rigid, while at large distances from the vortex center the rotational velocity profile decays with a hyperbolic dependence (Fig. 1). The asymptotic behavior of a Burgers vortex at small and large distances from the vortex center corresponds to a Rankine vortex [17,18]. The maximum rotational velocity in a Burgers vortex is $0.638\omega r_0$, which is attained at r/r_0 = 1.121. At a distance of r_{eff}/r_0 = 4.5 the rotation velocity is a third of the maximum value. We shall refer to this distance as the effective radius[1] of the vortex.

[1] In the literature, the effective radius of a Burgers vortex is taken to be 2.242(n/A), where n is the viscosity.

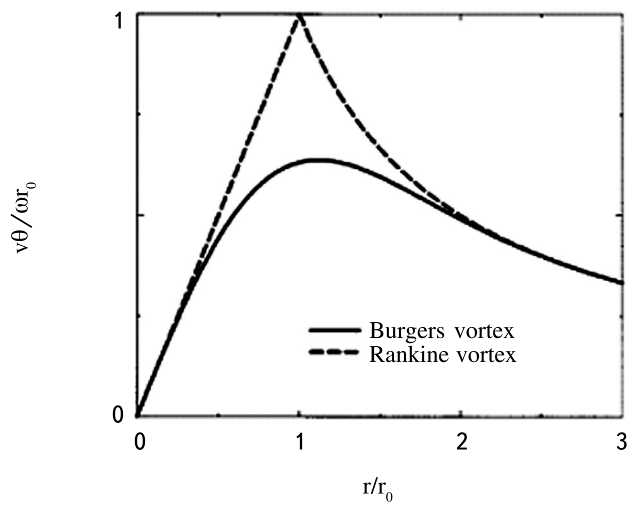

Fig. 1. Rotational velocity profiles in Burgers and Rankine vortices.

2. The order of magnitude of the parameters of the gas disk, solid particles, and vortex

We consider a Burgers vortex in a viscous, axially symmetric accretion disk with an effective temperature T, volume gas density r, and near-Kepler rotation. The speed of sound in the gas is estimated to be

$$c_s = \sqrt{\gamma kT/m_H} \approx (\gamma T/100\text{K})^{1/2}\ \text{km/s}, \tag{2}$$

where $\gamma = 1.4$ is the adiabatic index, k is the Boltzmann constant, and m_H is the mass of a hydrogen atom.

In the vertical direction, the gas is in hydrostatic equilibrium with a characteristic height scale of

$$H \sim \frac{c_s}{\Omega} \approx 0.03\left(\frac{T}{100\text{K}}\right)^{1/2}\left(\frac{M_\odot}{M}\right)^{1/2}\left(\frac{R}{\text{a.u.}}\right)^{3/2}\ \text{a.u.} \tag{3}$$

The surface density of the gas in the disk is estimated to be $\Sigma \approx 2H\rho$.

In the α-model for a disk [19], the gas flows at a velocity of $dm/dt = 3\pi\nu\Sigma$, where n is the kinematic viscosity of the gas with $\nu = \alpha c_s H$.

The characteristic dynamic (orbital) time scale for the disk is

$$\tau \sim \frac{1}{\Omega} \approx \frac{1}{5}\left(\frac{M_\odot}{M}\right)^{1/2}\left(\frac{R}{\text{a.u.}}\right)^{3/2}\ \text{year}. \tag{4}$$

In a Kepler disk the solution of the radial momentum equation leads to a difference between the velocities of the solid particles and the surrounding gas [20]. In a thin gas disk ($c_s << v_K \equiv \Omega R$), the solid particles drift toward the center at velocity relative to the gas of

$$\frac{\Delta \nu}{c_s} \sim \frac{c_s}{\Omega R} \approx 0.03 \left(\frac{T}{100\mathrm{K}} \right)^{1/2} \left(\frac{M_\odot}{M} \right)^{1/2} \left(\frac{R}{\mathrm{a.u.}} \right)^{1/2} . \tag{5}$$

For c_s~1 km/s the drift velocity is on the order of 30 m/s. The characteristic drift time scale [8,9] is almost two orders of magnitude longer than the dynamic time τ:

$$\tau_{\text{др}} \sim r/\Delta v \sim (R/\mathrm{a.e.}) 10^2 \text{ year}. \tag{6}$$

The solid particles precipitate (settle) on the plane of symmetry of the disk over a characteristic time [21]

$$\tau_{\mathrm{p}} \sim \Sigma / \alpha \Omega \rho^* , \tag{7}$$

where ρ^* is the mass density of the particles and the characteristic time between collisions of solid particles is estimated to be

$$\tau_{\mathrm{coll}} \sim D \rho^* / \Sigma^* \Omega , \tag{7'}$$

where D is the particle diameter, Σ^* is the surface density of particles in the disk, which is more than two orders of magnitude lower than S for the disk. For particles of meter size, this time is on the order of ~5 years. Thus, in order to create objects with sizes much greater than a meter, it is necessary to have a medium with a high particle concentration in comparatively small volumes. Vortices are such a medium.

Particles in the eddy of a vortex are subjected to centrifugal, Coriolis, and frictional forces and, to a lesser degree, to pressure gradient forces. While centrifugal force removes particles from the center of a vortex, frictional and Coriolis forces in an anticyclonic Burgers vortex are directed toward the center. In order to have the acceleration directed toward the vortex center, it is necessary that the angular circulation velocity $\omega = |d\theta/dt|$ of the gas in the vortex be less than 2Ω with low friction, i.e.,

$$\omega < 2\Omega \tag{8}$$

a condition that is always satisfied in practice.

The size of the vortex is limited by two processes: viscous dissipation and orbital shear. Viscous dissipation destroys vortices with sizes smaller than the viscous scale length [22]

$$L_\nu = \frac{\alpha c_s H}{v_\theta} \approx 0.003\left(\frac{\alpha}{0.01}\right)\left(\frac{0.1 c_s}{v_\theta}\right)\left(\frac{M_\odot}{M}\right)^{1/2}\left(\frac{R}{\text{a.u.}}\right)^{3/2} \text{a.u.}, \tag{9}$$

where v_θ is the rotational velocity of the vortex. But even if vortices are formed on this scale, they are much larger than the particles and can survive for many orbital periods.

The Kepler shear flow suppresses the formation of circular structures with sizes greater than the shear scale length [23],

$$L_{\text{sh}} = \sqrt{v_\theta \left|\frac{d\Omega}{dR}\right|^{-1}} \approx 0.05\left(\frac{v_\theta}{0.1 c_s}\right)^{1/2}\left(\frac{M_\odot}{M}\right)^{1/4}\left(\frac{R}{\text{a.u.}}\right)^{5/4} \text{a.u.} \tag{10}$$

Circular vortices with sizes greater than L_{sh} are stretched out in the azimuthal direction, so they can last longer. We have demonstrated [24] the possibility of the formation of an azimuthally elongated triaxial ellipsoidal vortex with a linear material velocity profile analogous to the *S* ellipsoids of Riemann [25] in a disk. We note, however, that in a disk surrounding a central object with a solar mass, at a distance of about 30 a.u., a vortex with a characteristic rotation velocity of $0.01c_s$ can have a circular shape and a characteristic size of ~1 a.u.

These estimates for L_ν and L_{sh} show that vortices are a large-scale phenomenon and that they are large enough to create planetesimals with sizes *L*>>*D*.

The dissipative frictional force to which solid particles are subjected by the gas in a gas disk is given either by the Stokes formula or the Einstein formula [15]. If the particle size *D* is small compared to the mean free path of the gas molecules, then these particles are subjected to the Einstein frictional force

$$F = \frac{\rho c_s}{\rho^* D}\Sigma^*(\mathbf{v} - \mathbf{u}), \tag{11}$$

where **v** is the velocity of the gas and **u** is the velocity of the particles. Larger particles are subjected to Stokes frictional force (see Eq. (15)).

3. Burgers vortex in the local reference system

We shall use the local approximation with a coordinate system that rotates with the disk at an angular velocity Ω_0 at a distance R_0 around a central star of mass *M*. In this approximation, assuming that the dimensions of the vortex are much less than the distance R_0, we choose a Cartesian coordinate system with center O (Fig. 2), the Y axis directed toward the star, and the X axis, along the material flow velocity. We express the rotation of the disk in the form $\Omega(R) \propto R^{-q}$. When only the gravitation of the central star is acting, the rotation will be Keplerian with q = 3/2, and for a uniformly rotating disk q = 2, i.e., $2 \geq q \geq 3/2$.

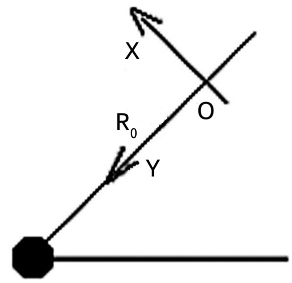

Fig. 2. The local coordinate system.

In this coordinate system, the material flow has an X component of the velocity equal to $\mathbf{i}\, q\,\Omega_0\, y$, the centrifugal inertial force is compensated by the radial component of the gravitation of the star at a distance R_0, and at other points their sum yields the tidal force $\mathbf{j}3\Omega_0^2\, y$. The vertical component of the gravitation, $\Omega_0^2\, z$, is the restorative force along the Z axis.

We begin by examining a gas disk surrounding a central object. In the local approximation, the equations for a time-independent isentropic shear flow of the gas including viscosity is

$$(\mathbf{v}\nabla)\mathbf{v} = \mathbf{j}3\Omega_0^2\, y - \mathbf{k}\Omega_0^2\, z - 2\mathbf{\Omega}_0 \times \mathbf{v} - \nabla h + \nu\Delta\mathbf{v} \tag{12}$$

$$\nabla(\rho\mathbf{v}) = 0\,, \tag{13}$$

where h is the specific enthalpy ($h = \int \rho^{-1}\, dp$) and **i**, **j**, and **k** are the cartesian unit vectors. The first term on the right of Eq. (12) is, as noted above, the tidal acceleration in the plane of the disk, the second term is the vertical component of gravitation, the third term is the Coriolis acceleration, and the fourth is the viscous stress term.

In the chosen coordinate system, the Burgers vortex has the form

$$\begin{aligned} v_x &= -Ax - \omega r_0^2\, y\left[1 - \exp\left(-r^2/r_0^2\right)\right]/r^2\,, \\ v_y &= -Ay + \omega r_0^2\, x\left[1 - \exp\left(-r^2/r_0^2\right)\right]/r^2\,, \\ v_z &= 2Az\,, \end{aligned} \tag{14}$$

where $r^2 \equiv x^2 + y^2$.

It is easy to confirm that the solution (14) satisfies the continuity equation $\rho = \text{const}$, which is acceptable in the local approximation.

On substituting Eq. (14) in Eq. (12) we obtain an expression for the specific enthalpy $h(x, y, z)$. We shall,

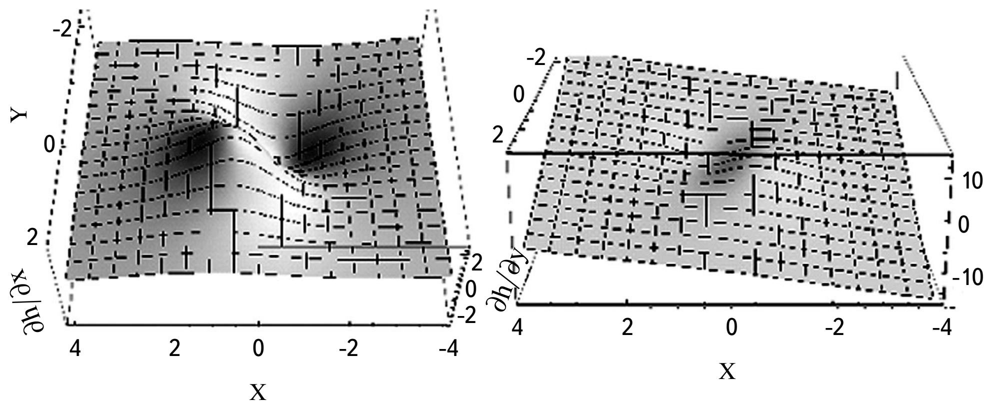

Fig. 3. The profile of the pressure gradient force ∇h, which makes the material in the disk rotate clockwise in an anticyclonic Burgers vortex.

however, omit the rather cumbersome expression for $h(x, y, z)$.. Here we show only the spatial profiles of the gradient force, $\partial h/\partial x$. and $\partial h/\partial y$, in the X,Y plane within an anticyclonic Burgers vortex (Fig. 3).

4. Dynamics of solid particles in a Burgers vortex

Initially we limit ourselves to studying the two-dimensional dynamics of solid dust particles in a Burgers vortex taking into account the pressure gradient force ∇h, tidal force, Coriolis forces, and friction. Here we neglect the effect of the solid particles on the dynamics of the gas, as well as the interactions of the particles among themselves.

We assume that the particle size D is much greater than the mean free path of the gas molecules, so that the friction of the solid particles with the gas is described by the Stokes force

$$\mathbf{f} = \beta(\mathbf{v} - \mathbf{u}), \quad (15)$$

where

$$\beta \equiv 18\rho \nu / \rho^* D^2 , \quad (16)$$

$\mathbf{u}$ is the particle velocity

$$\mathbf{u} = (dX/dt, dY/dt) \quad (17)$$

and X and Y are the particle coordinates.

The equation of motion for the dust particles in this approximation takes the form

$$du_x/dt = 2\Omega_0 u_y - \beta\left(\mathbf{v}_x\big|_{r=(X,Y)} - u_x\right) - \partial h/\partial x\big|_{r=(X,Y)}, \tag{18}$$

$$du_y/dt = 3\Omega_0^2 Y - 2\Omega_0 u_x - \beta\left(\mathbf{v}_y\big|_{r=(X,Y)} - u_y\right) - \partial h/\partial y\big|_{r=(X,Y)}. \tag{19}$$

It is convenient to represent these equations in dimensionless form. We take the size r_0 of the trunk of the vortex as the characteristic length of the problem, $1/\Omega_0$ as the characteristic time, and $\Omega_0 r_0$ as the characteristic velocity. Then Eqs. (18) and (19) become

$$du_x/dt = 2u_y + \gamma\left(v_x\big|_{r=(X,Y)} - u_x\right) - \partial h/\partial x\big|_{r=(X,Y)}, \tag{20}$$

$$du_y/dt = 3y - 2u_x + \gamma\left(v_y\big|_{r=(X,Y)} - u_y\right) - \partial h/\partial y\big|_{r=(X,Y)}, \tag{21}$$

where the dimensionless parameter γ is given by

$$\gamma = \beta/\Omega_0 = 18\rho\nu/\rho^* D^2 \Omega_0 . \tag{22}$$

We first examine the dynamics of the particles in the neighborhood of the vortex trunk ($r^2/r_0^2 < 1$), where the rotation profile of the vortex corresponds to solid rotation. In dimensionless form,

$$v_x = -Ax - \omega y + O\left(r^2/r_0^2\right), \quad v_y = -Ay + \omega x + O\left(r^2/r_0^2\right), \tag{23}$$

where A and ω are in units of Ω_0. From Eq. (2) with Eq. (13) we find

$$\partial h/\partial x = -\left(A^2 - \omega^2 - 2\omega\right)x - 2A(\omega+1)y, \tag{24}$$

$$\partial h/\partial y = 2A(\omega+1)x - \left(3 + A^2 - \omega^2 - 2\omega\right)y. \tag{25}$$

Substituting Eqs. (23)-(25) in Eqs. (20) and (21), we obtain the equations of motion for the solid particles in the neighborhood of the vortex trunk:

$$\begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{u}_x \\ \dot{u}_y \end{Bmatrix} = \begin{Bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a & b & -\gamma & 2 \\ -b & a & -2 & -\gamma \end{Bmatrix} \begin{Bmatrix} X \\ Y \\ u_x \\ u_y \end{Bmatrix}, \tag{26}$$

where

$$a = A(A-\gamma) - (\omega+1)^2 + 1; \quad b = 2A(\omega+1) - \gamma\omega. \tag{27}$$

It is clear from these equations that the equilibrium position of the solid particles in the vortex trunk is its center $X = Y = 0$, where $u_x = u_y = 0$ and $\dot{u}_x = \dot{u}_y = 0$. The particles in this region approach the center of the vortex along spiral paths.

In order to determine the stability of this equilibrium position, it is necessary that the real part of the eigenvalues of the matrix of coefficients (26) be equal to zero or negative.

The matrix (26) has complex eigenvalues of the form

$$\Lambda_{1,2,3,4} = -\gamma/2 \mp i \pm \sqrt{\left[a - 1 + \gamma^2/4 \pm i(b-\gamma)\right]}\ . \tag{28}$$

After separation of the real part, the stability condition becomes

$$(b-\gamma)^2 + \gamma^2(a-1) \le 0, \tag{29}$$

in which the centrifugal force is always less than the sum of the frictional and Coriolis forces, and the resultant force acting on a solid particle is directed toward the center of the vortex.

Equation (29) yields the necessary condition for stability: $a \le 1$. Together with Eq. (27), this gives $(\omega+1)^2 > A(A-\gamma)$, which is satisfied for arbitrary, positive A and γ. The condition (29), with Eq. (27), leads to the stability criterion

$$\gamma > A, \tag{30}$$

which, with Eq. (16), in dimensional form, gives the following for the viscosity ν:

$$\nu > \rho^* A D^2 / 18\rho. \tag{31}$$

We now consider whether or not an anticyclonic orbit exists over the entire volume of the Burgers vortex (1) on which the sum of the frictional and Coriolis forces is balanced by the centrifugal force, i.e., in the local cylindrical coordinate system

$$\gamma A r + 2u_\theta = u_\theta^2 / r, \quad \text{or} \quad (u_\theta / r)^2 - 2(u_\theta / r) - \gamma A = 0.$$

This is an transcendental equation, which has the dimensional form

$$\left[1 - \exp\left(-r^2/r_0^2\right)\right] r_0^2 / r^2 = B, \tag{32}$$

where

$$B \equiv \left[1 + \sqrt{\left(1 + \beta A / \Omega_0^2\right)}\right] \Omega_0 / \omega.$$

Equation (32) has a real solution for the radius of the orbit only for $B \le 1$ (see Fig. 4), with $r = 0$ for $B = 1$. It is clear from the expression for B that $B > 2\Omega_0/\omega$. The condition (8), on the other hand, yields $B > 1$. Thus, the only equilibrium position for the solid particles in the vortex is its center, where all the particles captured by the vortex, with a combined mass of

$$M^* \approx \pi r_{eff}^2 \Sigma^*, \tag{33}$$

are collected over a characteristic time

$$\tau \sim \omega r_{eff} / A\sqrt{\beta\nu}. \tag{34}$$

The mass of the solid particles captured by the vortex also forms a planetesimal.

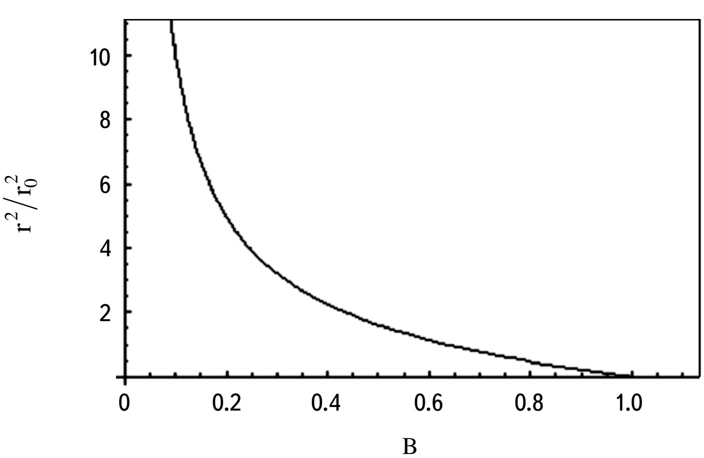

Fig. 4. The solution of the transcendental equation (32).

5. The thickness of the disk in the core region of the vortex

Up to now we have been considering the behavior of a Burgers vortex in the plane of the disk. However, a Burgers vortex is a three-dimensional structure. We now discuss the thickness of the disk in a region where the Burgers vortex is localized. To do this, we turn to the *z*-projection of the Navier-Stokes equation (12). Integrating this equation and using the formula for the velocity v_z, we obtain the dependence of the enthalpy on the coordinate *z*:

$$h(z) = c_{s0}^2 - \left(4A^2 + \Omega_0^2\right)z^2/2, \tag{35}$$

where c_{s0} is the speed of sound at the center of the vortex (the Clapeyron equation was used to estimate the enthalpy h_0 at the center of the vortex), and Ω_0 is the angular rotation velocity of the local coordinate system. This yields the half thickness of the disk in the region of the vortex core:

$$z_0 = c_{s0} \Big/ \sqrt{\left(2A^2 + \Omega_0^2/2\right)} \ . \tag{36}$$

The half thickness of a Kepler disk that contains no vortices is obtained using the hydrostatic equilibrium condition for the gas and is given by Eq. (3). The question arises of whether a Burgers vortex changes the thickness of the disk in the region where it is localized.[2]

At the radius R_0 the half thickness of the Kepler disk is

$$z_K \cong c_{s0}/2\Omega_0 \,. \tag{37}$$

The relative thickening of the disk is

$$\frac{\Delta z}{z_K} \equiv \frac{z_0}{z_K} - 1 = \frac{2\sqrt{2}}{\sqrt{\left(1 + 4A^2/\Omega_0^2\right)}} - 1, \tag{38}$$

which is positive for

$$A < 1.3\Omega_0 \,. \tag{39}$$

This condition is satisfied in all regions of a typical protoplanetary disk. Thus, a protoplanetary disk in the

[2]Vladimir Grinin brought this to my attention. For this I thank him.

localization region of a Burgers vortex is thicker.

6. Discussion

We now estimate the orders of magnitude of τ, M^* and $\Delta z/z_K$ for a model protoplanetary disk with radius 30 a.u. and mass $0.05\,M_\odot$ around a star with the sun's mass, $M \approx M_\odot$. The local reference system is set at a distance of $R_0 = 20$a.u. We then have $\Omega_0 \approx 2\cdot 10^{-9}$ s^{-1} and $\Sigma \sim 160\,$g/cm^2.

Since the temperature of the gas is on the order of $50 \div 150$ K, the sound speed is ≈ 1 km/s. Taking the maximum rotation velocity of the vortex at a distance $r_0 \approx 10^8$ km from its center to be 0.5 km/s and the velocity of the converging flow to be $v_r = A\cdot r_0 \approx 0.1\,$km/s, we obtain

$$\omega \approx 5\cdot 10^{-9}\ \text{s}^{-1}, \quad A \approx 10^{-9}\ \text{s}^{-1}. \qquad (40)$$

The stability condition for the position of solid particles at the center of a Burgers vortex for the viscosity (31) is satisfied with wide margins in the case of protoplanetary disks. In fact, the molecular viscosity of the gas estimated as $\nu \sim \lambda c_s$, where λ is the mean free path of the molecules and $\tilde{n}_s$ is the speed of sound, does not play a significant role in the processes taking place in a protoplanetary disk. In fact, the mean free path is defined as $\lambda \sim 1/n\sigma$, where n is the concentration of the gas molecules and σ is their interaction cross section. The molecular concentration in the central plane of the disk is on the order of $n \sim \Sigma/(2 m_H H) \approx 10^{14}$ cm^{-3}. Assuming that the interaction cross section is close to the area of a hydrogen molecule ($\sigma \sim 10^{-16}$ cm^2), we obtain $\lambda \sim 20$ cm and $\nu \sim 10^6$ cm^2/s. The characteristic time for evolution of the disk associated with these quantities, i.e., $\tau = R^2/\nu$, is on the order of ~10^{13} years, or 10^6 times the observed evolution time for a disk.

For this reason, the a-disk model [19] is usually used. In it, the turbulent character of the flows in accretion disks is actually taken into account and the turbulent viscosity is given by $\nu \sim \alpha c_s H \approx \alpha H^2 \Omega_0$, where the dimensionless parameter a is assumed constant with a value ranging from unity to ~10^{-2}. Here the viscous scale length $L_\nu \approx 10^5$ km, so that large vortices cannot be destroyed by viscosity. The Kepler shear length is $L_{\text{sh}} \approx 6\cdot 10^9$ km. Thus, vortices with sizes $r_{eff} < L_{sh}$ can have a circular shape.

Given that $\rho^*/\rho \approx 10^{10}$ in the plane of symmetry of the disk, $\Sigma^* \sim 2\,$g/cm^3, and using the average value for the viscosity from the vortex stability condition (31) and (16) in Eqs. (33) and (34), we obtain the estimates

$$M_3 \approx 10^{27}\ \text{g}; \quad \tau \sim 3\cdot 10^6 (\text{m/D})\,\text{years}.$$

Thus, over a time on the order of 10^6 years, for meter-sized particles, a mass comparable to the mass of the planet Venus will accumulate at the center of the vortex.

The relative thickening of the disk in the region where a Burgers vortex is localized is $\Delta z/z_K \cong 1$; that is, in the region of the core of the vortex, the local thickness of the disk is doubled!

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