Geodesic mapping of differetiable manifolds with torsion linear connection

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Introduction. The great investment of the theory of geodesic mapping of Riemannian spaces has been made by T. Levi-Chivita, H. Weyl, T.Y.Thomas, and N.S.Sinyukov [1]. The geogesic mapping of Tangent bundles of differentiable manifolds has been studied by me [2]. In the present paper we consider the Geodesic mapping of differetiable manifolds with torsion linear connection [2], pp.61-68.

Section 1. Let M and $\widetilde{M}$ are differentiable manifolds, $dimM = dim\widetilde{M} = n$, ∇ and $\widetilde{\nabla}$ are linear connections on M and $\widetilde{M}$ respectively, $S(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y]$ and $\widetilde{S}(\widetilde{X},\widetilde{Y}) = \widetilde{\nabla}_{\widetilde{X}}\widetilde{Y} - \widetilde{\nabla}_{\widetilde{Y}}\widetilde{X} - [\widetilde{X},\widetilde{Y}]$ their torsion tensors, $f : M \to \widetilde{M}$ diffeomorphism.

Linear connections ∇ and $\widetilde{\nabla}$ have equal torsion if $S(X,Y) = \widetilde{S}(\widetilde{X},\widetilde{Y})$ where $\widetilde{X} = df(X), \widetilde{Y} = df(Y)$.

$f : M \to \widetilde{M}$ diffeomorphism is a geodesic mapping, if and only if the linear connections on M and $\widetilde{M}$ have equal torsion and there exist covector φ_i such that $\widehat{\nabla}_k f^i_j = f^i_j\varphi_k + f^i_k\varphi_j$, where $\widehat{\nabla}_k$ is the mixed covariant derivative with respect to $(\nabla, \widetilde{\nabla})$.

References

[1] N.S.. Sinyukov. Geodesic mapping of Riemannian space. *Moscow. Nauka.*, 1979.

[2] V.A.. Piliposyan. Geodesic mapping of Tangent bundles. *Ph.d.Dissertation.in Math. 01.01.04. Kasan.*, 1987.