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Impurity-related intraband absorption in coupled quantum dot-ring structure under lateral electric field



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ABSTRACT

The effects of a lateral electric field on intraband absorption in *GaAs/GaAlAs* two-dimensional coupled quantum dot-ring structure with an on-center hydrogenic donor impurity are investigated. The confining potential of the system consists of two parabolas with various confinement energies. The calculations are made using the exact diagonalization technique. A selection rule for intraband transitions was found for x-polarized incident light. The absorption spectrum mainly exhibits a redshift with the increment of electric field strength. On the other hand, the absorption spectrum can exhibit either a blue- or redshift depending on the values of confinement energies of dot and ring. Additionally, electric field changes the energetic shift direction influenced by the variation of barrier thickness of the structure.

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1. Introduction

Infrared photodetectors have been the subject of intensive experimental and theoretical research during the last few decades [1]. Such photodetectors are based on either intrinsic free carriers, impurity absorptions of incident light or on intersubband photoabsorption in low-dimensional systems, like quantum wells (QW), dots and rings. An attractive advantage of using QWs [2], quantum dot (QD) [3] and quantum ring (QR) [4] photodetectors is the reduced dimensionality of these systems, which yields a narrower photoresponse. Additionally, QR and QD photodetectors are sensitive to light polarization parallel and perpendicular to the growth axis (*z*), while QW photodetectors are only affected by *z*-directed light polarization. This polarization dependence gives QR and QD photodetectors an advantage over QW photodetectors. These optical characteristics have recently initiated a great interest to investigate theoretically the intraband optical properties of semiconductor QDs and QRs, taking into account the influence of external electromagnetic fields [5–15].

Additionally, the impurity problem in QRs and QDs is also of interest and helps understand the optical properties of these structures. Impurity related phenomena, such as donor binding energy, electronic structure in external electric and magnetic fields

and photoionization cross section, lead to numerous controlling possibilities of physical properties of optical devices based on QR and QD structures [16,17]. Many works have been devoted to the theoretical investigation of these phenomena [18–28].

Recently, there has been an increasing demand for the development of complex quantum confined systems [29] for both practical applications and fundamental studies, which include geometrical quantum phases [30], spin–spin interactions [31] and quantum state couplings [32]. Theoretical investigations indicate that changes in the nanostructure's shape are accompanied by an alteration of the total spin of the ground state [31,33], which could lead to the design of tunneling spin switches if dot-ring coupled nanostructures. Recently a *GaAs/AlGaAs* laterally coupled quantum dot-ring (CQDR) nanostructure has been developed by Somaschini et al. [34].

Although the physical properties of novel CQDR structure have not been investigated theoretically on large scales, some of the following articles are worth mentioning. Few-electron systems confined in CQDR in the presence of an external magnetic field have been studied by the exact diagonalization technique [35]. The authors investigated the distribution of electrons between the QD and the QR. It has been shown that this distribution depends not only on the parameters of the confinement potential but it can also be altered by external magnetic fields. Linear and nonlinear optical susceptibilities in CQDR have been theoretically studied in [36], where a model for the potential, that assumes parabolic confinement in both the QD and the QR, is adopted. One-electron energies

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and wave functions have been found using the potential morphing method in the framework of effective mass approximation. It was found that the electron ground state can be changed from a nearly pure dot-localized state to a nearly pure ring-localized state or vice versa by suitably choosing the structure parameters, i.e. the ring (or dot) confinement energy and/or the depth of the dot confinement relative to the bottom of the ring potential. In calculations by Zipper et al., they have shown that the manipulation of confinement parameters can alter the overlap of the electron wave functions, so that the transition probability will be enhanced or suppressed on demand [37]. The systematic studies of such manipulations' influence on relaxation times, optical absorption and conducting properties of CQDR have been done as well. Also, recently Barseghyan made a detailed investigation of the lateral electric field effect on one-electron states in CQDR structure for cases with and without an on-center hydrogenic donor impurity [38]. The results show that the probability density of the electron ground state is drastically influenced by the confinement energies, depth of the dot, barrier thickness and hydrogenic impurity. Additionally, the influence of lateral electric fields on the energy levels strongly depends on the electron localization type (ring- or dot-localized state). The effects of an externally applied electric field on the linear optical absorption and relative refractive index change associated with transitions between off-center donor impurity states in CQDR structures have been reported in [39]. A significant sensitivity reported by Correa et al., is produced by the geometrical distribution of confining energies, as well as to the strength of the applied field.

The objective of this study is to investigate the effect of a lateral electric field on the impurity-related intraband absorption spectrum in a CQDR. The paper is organized as follows: in Section 2 we present the theoretical framework. Section 3 is dedicated to the results and discussion. Finally, the conclusions are given in Section 4.

2. Theoretical framework

The Hamiltonian of the GaAs CQDR system is written in effective mass and parabolic band approximations, considering an electron-impurity Coulomb interaction and the influence of lateral electric field:

$$\hat{H} = -\frac{\hbar^2}{2m^*}\nabla^2 + V(\rho) + eF\rho \cos \varphi - \frac{e^2}{\epsilon\rho}, \quad (1)$$

where m^* is the electron effective mass, e is the unit electronic charge and ϵ is the dielectric constant of the considered material. We choose the electric field oriented along the x -axis. Here we model a strictly two-dimensional cylindrically symmetric potential of a QD placed within the QR with the following confinement:

$$V(\rho) = \min[m^*\omega_d^2\rho^2/2, m^*\omega_r^2(\rho - R)^2/2], \quad (2)$$

where $\hbar\omega_d$ and $\hbar\omega_r$ are the confinement energies of the dot and ring. The radius of ring R is defined by the sum of oscillator lengths for the dot and ring related wells and the barrier thickness between dot and ring d according to $R = \sqrt{2\hbar/m^*\omega_d} + \sqrt{2\hbar/m^*\omega_r} + d$. This confinement potential model has been used in [35,36,38,39] and a similar potential in laterally coupled double QDs has been used as well in [40–43]. We calculated the one-electron eigenvalues and eigenfunctions solving two-dimensional Schrödinger equation via the exact diagonalization technique [44,45].

Calculations of the intraband optical absorption are based on Fermi's golden rule with the corresponding coefficient [15]:

$$\alpha(\hbar\omega) = \frac{16\pi^2\beta_{FS}\hbar\omega}{n_r V} N_{if} |M_{fi}|^2 \delta(E_f - E_i - \hbar\omega), \quad (3)$$

where n_r is the refractive index of GaAs, V is the volume of the sample per CQDR (in this work $V = 648 \times 10^{-18} \text{ cm}^3$ [34]), β_{FS} is the fine structure constant, $\hbar\omega$ is the incident photon energy, E_f and E_i are the energies of the final and initial states, respectively. $N_{if} = N_i - N_f$ is the difference of the numbers of electrons in the initial and final states and since we consider only a one particle problem, $N_i = 1$ must be taken for the ground state and $N_f = 0$ for all upper states. M_{fi} is the matrix element of coordinate. A y -polarization of incident light is considered and the δ -function is substituted by a Lorentzian profile with a full width at half maximum of 0.8 meV.

3. Results and discussion

The calculations are performed for GaAs material with parameter values $n_r = 3.6$, $m^* = 0.067 m_0$, where m_0 is the free-electron mass.

It is well known, that the intraband optical properties of semiconductor nanostructures strongly depend on the symmetry of initial and final states of the electron and on the orientation of incident light polarization. In this aspect, it is necessary to show how the symmetry of the electron wave functions changes in the presence of a donor impurity and oriented along the x -axis electric field. In the absence of the field and in the presence of on-center two-dimensional donor impurity, all the states with $l=0$ (angular momentum quantum number) will be non-degenerated, whereas the states with $l \neq 0$ will be twice degenerated which is a result of the axial symmetry of the structure. Influence of x -axis oriented lateral electric field destroys this symmetry, but it also creates a symmetry with respect to the x -axis, so there is a definite parity of the wave functions in the y variable. The first, second, and the forth states are even, while the third state is an odd wave function of y variable: $\Phi_{1,2,4}(x, -y) = \Phi_{1,2,4}(x, y)$, $\Phi_3(x, -y) = -\Phi_3(x, y)$ (for simplicity the wave functions are written in Cartesian coordinates). This fact will help us to understand the selection rule of the considered transitions.

In Fig. 1 we present the dependencies of the threshold energies of the optical transitions from the ground $N=1$ to the $N=2, 3, 4$ excited states on the electric field strength, where N labels the bound electronic states in an increasing manner of the energy. The results are for $d=10$ nm and different confinement energies of the dot and ring. In both figures we see that the curves for $1 \rightarrow 3$ and $1 \rightarrow 4$ transitions start from the same point, which contrasts the $1 \rightarrow 2$ transition case. The explanation for this is based on the fact that in the absence of electric field $N=1$ and $N=2$ states have $l=0$ and are not degenerated, while states with $N=3$ and $N=4$ have $l \neq 0$ and are twice degenerated. The influence of the electric field removes this degeneracy.

Also, the ground state energy is the lowest lying energy and “feels” the presence of impurity in the strongest way compared to the excited states. Impurity makes ground state energy level almost non-affected by electric field, causing the threshold energies to vary only because of the changes in excited energy levels.

The lowest excited state, $N=2$, thus is most affected by the tilting of the confining potential caused by the electric field. This makes E_{21} to show the greatest variation. All the threshold energies decrease in function of the electric field, except the E_{41} , that is the only one that shows an increment with electric field, as a consequence of the complex behavior of the excited energy levels with the field (eigenvalues of the CQDR system are well discussed in Ref. [38].)

Additionally, the effect of confinement energy variation on

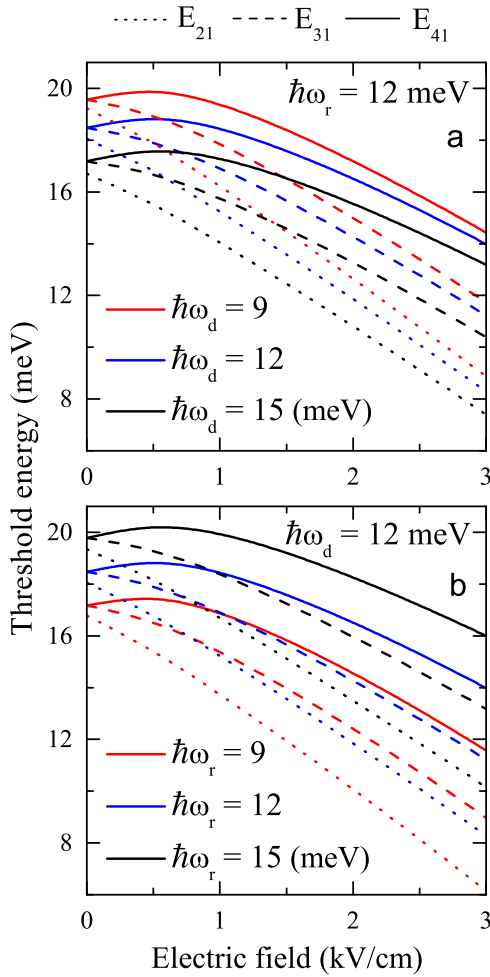


Fig. 1. Dependences of threshold energies for transitions from the ground $N=1$ state to $N=2, 3, 4$ on the electric field strength. Various values of confinement energies for dot (Fig. 1(a)) and ring (Fig. 1(b)) geometries are considered. The barrier thickness is $d=10$ nm.

threshold energies depends on region of the potential that we deal with, i.e. QD or QR region. If we strengthen the confinement in the QD region (increment of $\hbar\omega_d$) it will result in an increment of effective height of the confining potential around the QD and the potential will be more constricted as well (in accordance with Eq. (2)). The latter modifications of confining potential have a strong influence on the ground state, given that it has the lowest lying level that goes up. The impact on excited states is weaker, energies increase as well, but slower, and as a consequence they have a decreasing threshold energies (Fig. 1(a)).

The opposite effect is observed in the case of an increment of $\hbar\omega_r$, which yields an increment of the effective potential's height around the QD, while the width remains constant. These modifications are "felt" in a stronger way by the excited energies which increase faster than that of the ground state, and in this case, give way to increasing threshold energies (Fig. 1(b)).

Fig. 2 shows the dependences of threshold energies on electric field strength for different barrier thicknesses and fixed $\hbar\omega_d = \hbar\omega_r = 12$ meV. Here, like in Fig. 1, we again see that only the threshold energy for the $1 \rightarrow 4$ transition exhibits both an increase and a decrease with respect to electric field strength increment, while E_{21} and E_{31} only diminish. One can also observe, depending on the electric field strength range, that the barrier thickness variation has diverse influences on threshold energies. In the interval from 0 to approximately 1 kV/cm of electric field strength, an increment of d increases the threshold energies. Above 1 kV/cm

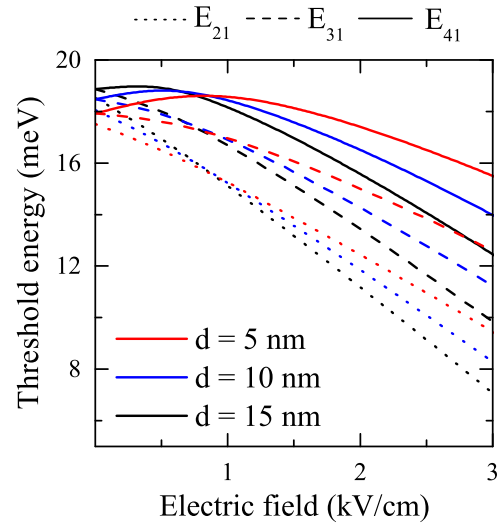


Fig. 2. Dependences of threshold energies for transitions from the ground $N=1$ state to $N=2, 3, 4$ on the electric field strength. Various values of the barrier thickness of CQDR are considered. The results are for $\hbar\omega_d = \hbar\omega_r = 12$ meV.

the opposite effect is observed.

To understand this, at first, it should be pointed out that the presence of an impurity makes the ground state energy almost unaffected by changes in d , as in the case of variations in $\hbar\omega_d$ and $\hbar\omega_r$. The increment of d results in the enlargement of the effective well width in the dot region, because it increases the radius R of the ring. As a result, a larger part of the electron cloud will be shifted to the dot region and the excited energy levels will go up as a consequence of strengthened confinement. Here, the tilting of the confining potential produced by electric field strength $[0, 1$ kV/cm] is not large, i.e. the impurity has only a considerable effect on the ground state.

Meanwhile, strengths larger than 1 kV/cm can tilt the confining potential in such a way that an enlargement of the well around the dot produces a shift of electronic cloud away from the dot region, and the increment of d will reduce the excited energies. The influence of the electric field on the variation of barrier thickness is responsible for the coupling of states in the CQDR structure.

In Fig. 3 we present the intraband optical absorption coefficient as a function of incident photon energy for several values of confinement energies of the dot (Fig. 3(a)), ring (Fig. 3(b)), electric field strength and fixed barrier thickness. Considering that incident light polarization direction is taken along the x -axis, the selection rule will allow transitions only from the ground $N=1$ (even) to $N=3$ (odd) state. The absorption coefficient of these transitions is shown in Fig. 3. According to the results in Fig. 1, increasing of electric field brings only a redshift in the absorption spectrum for the $1 \rightarrow 3$ transition, while changes in confinement energies may yield both a blue- or red- shift: an increment of $\hbar\omega_d$ results in a redshift and an increment in $\hbar\omega_r$ in blueshift. In addition, in the beginning of electric field variation the maximum of absorption coefficient goes up and later decreases. This tells us that the interaction between the incident radiation field and the electron strengthens and weakens correspondingly. In Fig. 4 the result for absorption coefficients for fixed $\hbar\omega_d = \hbar\omega_r = 12$ meV is presented for different values of barrier thickness and electric field strength. Here a redshift in absorption spectrum and a non-monotonic variation of maximum is observed as a result of electric field strength increment, like it was in Fig. 1.

Moreover, a redshift occurs if d is increased while the field strength is kept in the $[0, 1$ kV/cm] interval, while a blueshift is observed within the $[1, 3]$ kV/cm range. This effect is explained with the threshold energies in Fig. 2. Finally, the absorption maximum reduces considerably with increasing d .

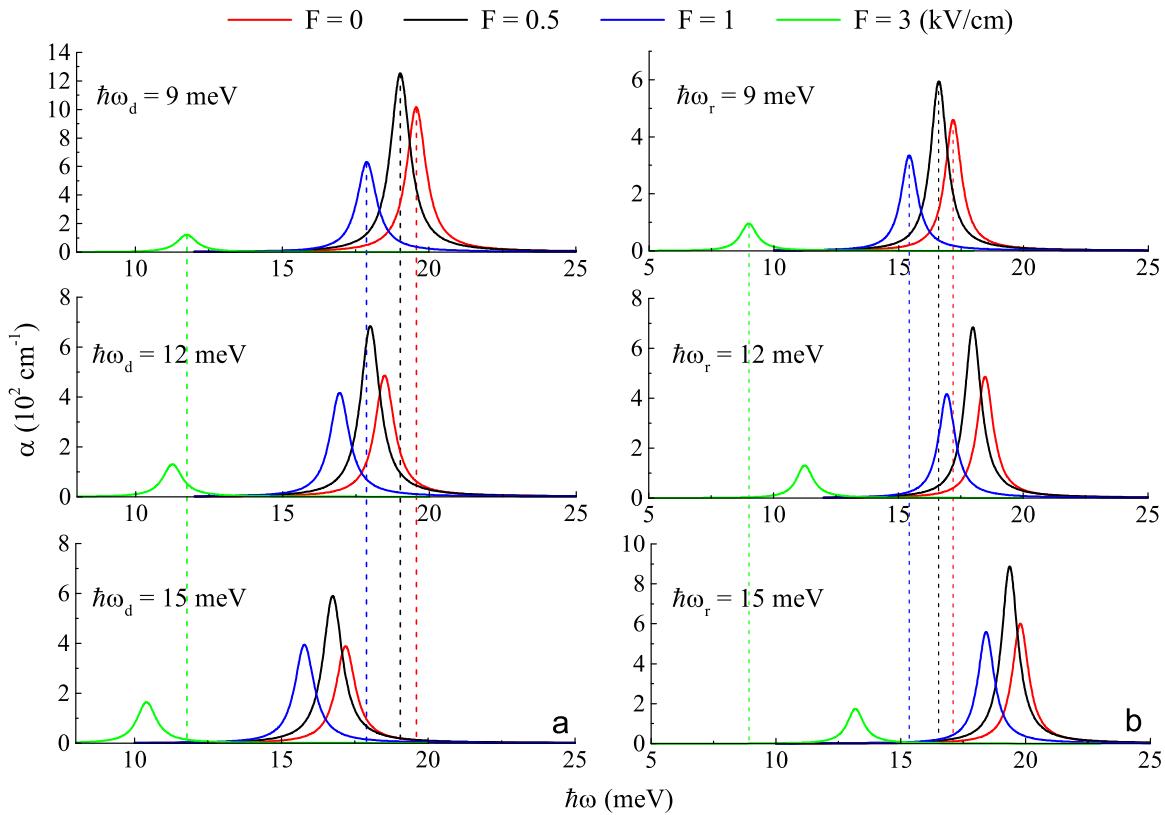


Fig. 3. Dependence of intraband optical absorption coefficient on incident photon energy in CQDR. The results are presented for $d = 10$ nm and $\hbar\omega_r = 12$ meV. Several values of dot confinement energy (Fig. 3(a)) and ring confinement energy (Fig. 3(b)) electric field strength are considered.

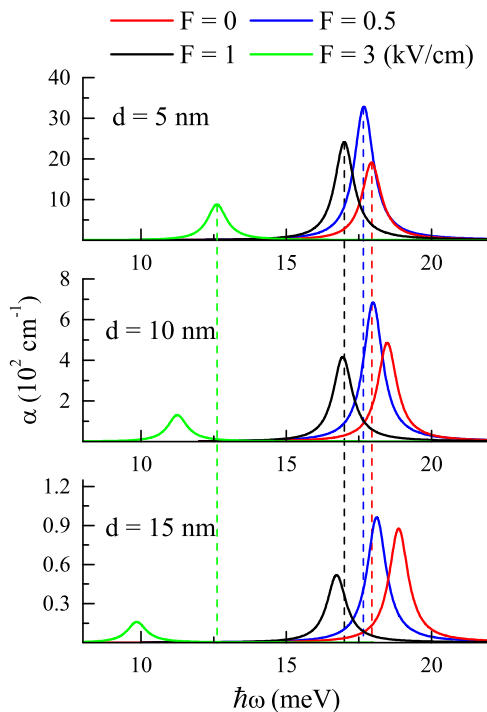


Fig. 4. Dependence of the intraband optical absorption coefficient on incident photon energy in CQDR. The results are presented for $\hbar\omega_r = \hbar\omega_d = 12$ meV. Several values of the barrier thickness of the structure and electric field strength F have been considered.

4. Conclusions

We have considered coupled dot-ring structure doped with a single on-center hydrogenic impurity under the influence of a

lateral electric field. Although threshold energies showed a dependence on electric field strength, exhibiting both increasing and decreasing variations, it turns out that only the $1 \rightarrow 3$ transition is feasible with the considered x -polarized incident light and that its related absorption spectrum exhibits only a blueshift.

This selection rule is a result of the symmetric modification of its parabolic confining potential created by the electric field. Dot-related confinement energy increments yield a redshift, while that of the ring produces a blueshift of the absorption spectrum. Additionally, we have found strong couplings between electric field and dot (or ring) barrier distance on the spectrum shifts: for the fields with strength up to 1 kV/cm increment of barrier distance results in the blueshift, while for the strength $F \geq 1$ kV/cm a redshift is observed.

We hope, that the results obtained within this work, regarding the dependence of intraband absorption phenomena on hydrogenic impurities, electric field strengths and confinement potential characteristics, contribute to the understanding of optical properties of coupled nanostructures with geometries similar to dot-ring structure.

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