PROCEEDINGS OF SCIENCE

Magnetic field of strange stars with rotating superfluid core

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The generation of a magnetic field and its distribution inside a rotating strange star are discussed. The difference between the angular velocities of the superfluid and superconducting quark core and of the normal electron plasma increases because of spin-down of the star and this leads to the generation of a magnetic field. The magnetic field distribution in a star is found for a stationary value of difference of angular velocities of these components. In all parts of the star this field is determined entirely by the total magnetic moment $\mathfrak{M}$ of the star which can vary from 10^{31} to 10^{34} G cm^3 depending on the model of compact star. We also estimate the maximum possible values of magnetic field on the surfaces of various models of strange dwarfs. We find that depending on configuration parameters - the mass M and the radius R - the upper limit is in the range 10^3-10^5 G. Such values of magnetic field may be an additional distinguishing feature that identifies the strange dwarfs within the larger class of observed white dwarfs.

The Modern Physics of Compact Stars 2015
30 September 2015 - 3 October 2015
Yerevan, Armenia

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PoS(MPCS2015)017

1. Introduction

The discovery of new classes of compact stars with magnetic fields of the order of $10^{14} - 10^{15}$ G, i.e., magnetars, anomalous x-ray pulsars (AXP), and soft gamma repeaters (SGR) [1], as well as heavy neutron stars with masses of the order of $2M_\odot$ [2, 3], has provided motivation for new studies of the properties of matter at supranuclear densities, in particular the possibility of formation of quark matter phases. The theories of strong interaction predict a phase transition from baryonic matter to quark plasma at densities several times the nuclear saturation density. In Refs. [4–7] the possibility of the appearance of strange quark matter where the s-flavor quarks coexist in equal amounts with the light flavor u and d quarks was discussed. It was argued that such state may be energetically more favorable than non-strange quark matter. Strange quark matter can form self-sustaining bound states in the form of "strange quark stars" (SQS) even in the absence of gravitation. Such an object may also constitute the core of an ordinary neutron star or a white dwarf.

Quark matter may be in a normal (unpaired) state as well as in a superfluid and/or superconducting state. Above a critical temperature $T_c \approx 50$ MeV and at densities characteristic for dense compact stars u, d and s quarks do not form Cooper pairs, i.e., the quark matter is normal. For temperatures $T < T_c$, however, quark matter becomes a superfluid and/or superconductor because the attraction between quarks leads to formation of Cooper pairs. Model calculation show that at densities of the order of $2\rho_0$ (where ρ_0 is the nuclear saturation density) quark matter is in the 2SC phase [8], where only u and d flavor quarks of two colors are paired. In this case an electron plasma must be present throughout the entire quark matter in order to ensure charge neutrality. For densities $\rho \gg \rho_0$, however, the CFL phase of quark matter is energetically more favorable than the 2SC phase [9]. Under equilibrium with respect to weak interactions and charge neutrality the matter of SQS consists of equal amounts of u, d and s quarks and electrons are absent. Under these conditions the pairing corresponds to the CFL phase in which massless u, d, and s quarks of all three colors pair [10, 11]. In the above mentioned models for SQS, the density of s quarks decreases towards the boundary of the quark core and this leads to emergence of electrons which are required to maintain overall charge neutrality [12–14]. Since electrons are bound to the quark core only by the Coulomb interaction, they may abandon the quark surface and form an electron plasma having a thickness of the order of $10^2 - 10^3$ fm. For this reason, a thin charged layer appears at the surface of a strange quark star, where the electric field intensity attains values of $10^{17} - 10^{18}$ V/cm [15–17]. The electric field in the near-surface charged layer is directed outward. Consequently, it may support a crust that consists of atomic nuclei and degenerate electrons (the Ae phase). The Ae phase cannot exist in chemical equilibrium with strange quark matter and, therefore, is bound to quark matter core only by gravity. A strange quark star may acquire a crust during the collapse of a supernova [18, 19] or as a result of matter accretion [20, 21]. Since free neutrons that have no electric charge can pass through the electrostatic barrier without hindrance and can be absorbed by the strange quark matter, the maximum density of the crust is limited to the density of formation of neutron drops $\rho_{\rm drip} \sim 4.3 \cdot 10^{11}$ g/cm^3.

For strange stars with masses $M > 0.5\ M_\odot$, the crust's thickness and mass are negligible compared to the star's radius and mass [22]. If the mass of a strange star satisfies the condition $M < 0.02\ M_\odot$, then the crust swells significantly and its maximum radius is of the order of that of white dwarfs. Unlike ordinary white dwarfs, these configurations, which are referred to as "strange dwarfs", have a core consisting of a strange star, which has a small size and low mass. The family of strange dwarfs is conventionally divided into two distinct categories [23]. The first one consists of a core of strange matter enveloped within ordinary white dwarf matter with density up to $\rho \sim 10^9$ g/cm^3. Such stars are hydrostatically stable with or without the strange core. The stars of second category contain superdense nuclear material with density up to $\rho \sim 4.3 \cdot 10^{11}$ g/cm^3 and they owe their hydrostatic stability to the strange quark matter core. Below we model such objects assuming that the nuclear matter envelope surrounding the quark core has density in the range $10^9 \leq \rho < 4 \cdot 10^{11}$ g/cm^3. This allows us to take into account both possibile categories of strange dwarfs.

The generation of a magnetic field in a quark star, with the star's rotation taken into account, has been studied [14] under the assumption that the matter in the star is in the normal state. It was shown that if the angular velocity Ω_+ of quark matter differs from that of the electrons, Ω_-, then a magnetic field will be generated that is uniform inside and dipolar outside the quark star. The acquired stellar magnetic moment is proportional to the difference $\Omega_+ - \Omega_-$ of the angular velocities, but the reason for the possible difference between Ω_- and Ω_+ was not discussed.

In the present article we refine the mechanism for generation of magnetic field in a rotating SQS taking into account the superfluidity and superconductivity of quark matter. We shall see below that this leads naturally to the difference $\Delta\Omega = \Omega_s - \Omega_n$ in the angular velocities Ω_s of the superfluid quark matter and Ω_n of the normal electron plasma, which is required to generate a magnetic field. Section 2 considers the distribution of the electric field in the near-surface layer of the quark core. Section 3 discusses the possibility of differential rotation of the superfluid and normal quark star components. In Section 4 we find the distribution of the magnetic field of a quark star with a crust. Finally in Section 5 we present several quark star models, determine the values for their magnetic fields, and discuss the feasibility of detecting quark stars as magnetars or white dwarfs.

2. Electric field at the surface of a strange quark core

As shown in Ref. [14], the differential rotation of the positively charged quark core and the electron layer linked to the crust results in the appearance of a surface current

$$i = \frac{\sigma}{2\pi}\left(\Omega_s - \Omega_n\right), \tag{2.1}$$

where

$$\sigma = \frac{E}{4\pi} \tag{2.2}$$

is the surface charge density, and E is the radial electric field at the surface of the quark core. Positive charges at the quark core surface are distributed within a layer of thickness of the order of 15 fm [24], which is the effective range of the strong interaction. However, following Ref. [15] we shall consider below a simple model, in which the charge of the quark core is taken to be uniformly distributed over the entire volume of the star (the Thomas-Fermi model). Then the charge density in the quark core is given by

$$\rho_{\rm core} = \frac{e}{3}\left(2n_u - n_d - n_s\right) - en_e, \tag{2.3}$$

where n_u, n_d, n_s and n_e are, respectively, the density of u, d, and s quarks and electrons. The electron density is found from the equation

$$n_e = \frac{p_e^3}{3\pi^2\hbar^3}, \tag{2.4}$$

where p_e is the electron Fermi-momentum which, in turn, is found from the requirement of mechanical equilibrium. In the case of ultrarelativistic electrons this requirement reads

$$\mu_\infty = p_e c - e\varphi = \text{const.} \tag{2.5}$$

where φ is the electrostatic potential. At large distances from the star $\varphi \to 0$ and $n_e \to 0$, therefore $\mu_\infty = 0$ and consequently

$$p_e = \frac{e}{c}\varphi. \tag{2.6}$$

We consider a two-dimensional geometry of a quark star, in which the quark core occupies the half-space $z \le 0$, the electron layer, the region $0 < z \le z_e$, and the star crust, the half-space $z > z_e$. In order to determine the electric field at the surface of the quark core, we first solve the Poisson equation for the potential φ in each of these regions. In the present geometry the Poisson equation takes the form

$$\frac{d^2\varphi}{dz^2} = -4\pi \begin{cases} \rho_{\rm core} = \dfrac{4\alpha^2}{3\pi}\dfrac{1}{\hbar c}\left(\varphi^3 - \varphi_q^3\right), & z \le 0, \\ \rho_{\rm el} = \dfrac{4\alpha^2}{3\pi}\dfrac{1}{\hbar c}\varphi^3, & 0 < z \le z_e, \\ \rho_{\rm crust} = \dfrac{4\alpha^2}{3\pi}\dfrac{1}{\hbar c}\left(\varphi^3 - \varphi_{cr}^3\right), & z > z_e, \end{cases} \tag{2.7}$$

where $\alpha = e^2/\hbar c = 1/137$ is the fine structure constant, $\rho_{\rm core}$, $\rho_{\rm el}$, $\rho_{\rm crust}$ are the charge densities in the quark core, electron layer, and star crust. We used expressions (2.3) - (2.6) to obtain equation (2.7). The potentials φ_q and φ_{cr} are defined via the density of quarks n_u, n_d, n_s in the core and ions n_i in the crust, respectively, in the following manner:

$$\varphi_q^3 = \pi^2\left(\frac{\hbar c}{e}\right)^3 (2n_u - n_d - n_s) \approx 3\pi^2\left(\frac{\hbar c}{e}\right)^3 n_e, \tag{2.8}$$

$$\varphi_{cr}^3 = 3\pi^2\left(\frac{\hbar c}{e}\right)^3 (Zen_i) \approx 3\pi^2\left(\frac{\hbar c}{e}\right)^3 n_e. \tag{2.9}$$

Note that $e\varphi_q$ and $e\varphi_{cr}$ are, in fact, the Fermi momenta of electrons in the quark core and at the base of the crust, respectively. Consequently, the boundary conditions on Eqs. (2.7) are

$$\begin{gathered} \varphi\left(z \to -\infty\right) = \varphi_q, \quad \varphi\left(z \to +\infty\right) = \varphi_{cr}, \quad \varphi|_{z=-0} = \varphi|_{z=+0}, \\ \left.\frac{d\varphi}{dz}\right|_{z=-0} = \left.\frac{d\varphi}{dz}\right|_{z=+0}, \quad \varphi|_{z=z_e-0} = \varphi|_{z=z_e+0}, \quad \left.\frac{d\varphi}{dz}\right|_{z=z_e-0} = \left.\frac{d\varphi}{dz}\right|_{z=z+0}. \end{gathered} \tag{2.10}$$

The magnitudes of $e\varphi_q$ and $e\varphi_{cr}$ can be obtained directly if the electron density in the strange quark core and in the star's crust is known. Equations of state of strange quark matter derived in Ref. [25] predict electron density to be about 10^{-5} -10^{-4} of the baryon density. Then, in parallel to Ref. [15], we may take $e\varphi_q \approx 20$ MeV. The electron density in the crust of a strange star is obtained in manner analogous to ordinary neutron stars by imposing charge neutrality and β-equilibrium among the constituents of crustal material [26]. In the density range $10^9 \leq \rho < 4 \cdot 10^{11}$g/cm^3 the value of the potential in the crust is $4 \leq e\varphi_{cr} \leq 10$ MeV. Fig. 1 shows the result of numerical integration of equations (2.7) with boundary conditions (2). The width of the electron layer between the quark core and the crust of the star is of the order of $z_e \sim 10^2 - 10^3$fm. The rapid change of the potential

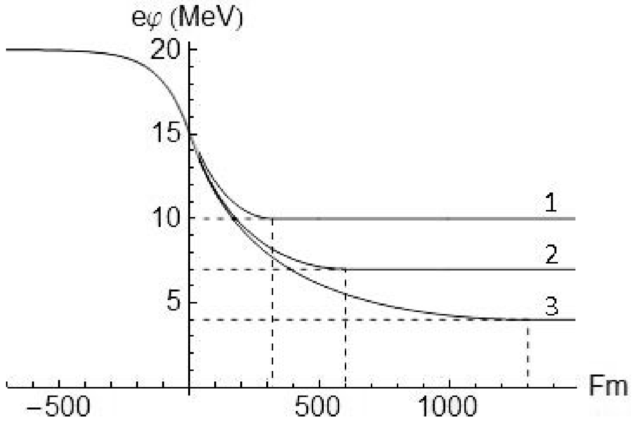

Figure 1: The potential $e\varphi(z)$ as a function of distance to the quark core for different values of potential in the crust: (1) $e\varphi_{cr} = 10$ MeV, (2) $e\varphi_{cr} = 7$ MeV, (3) $e\varphi_{cr} = 4$ MeV.

in the narrow electron layer induces an electric field of the order of $5 \cdot 10^{17}$ V/cm in that layer. The dependence of this electric field on the distance from the quark core is shown in Fig. 2. As shown

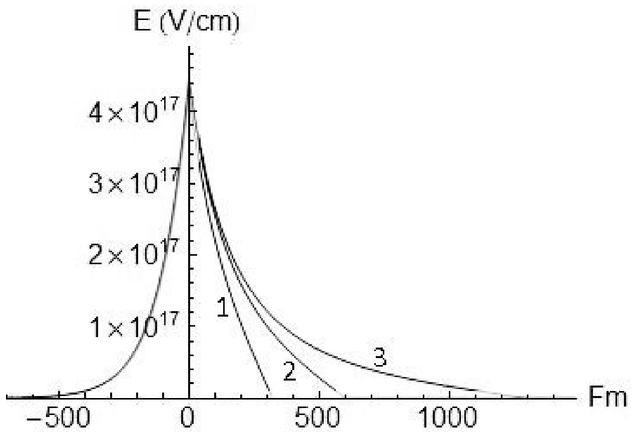

Figure 2: Electric field $E(z)$ as a function of distance to the quark core for difference values of electron energy in the crust: (1) $e\varphi_{cr} = 10$ MeV, (2) $e\varphi_{cr} = 7$ MeV, (3) $e\varphi_{cr} = 4$ MeV.

in Refs. [17,24] the electric field can rise up to the order of 10^{18} V/cm due to the β decay of quarks near the surface of a CFL quark matter core. Given a value for the electric field at the surface of the quark core, we can calculate the surface charge density and the surface current density using Eqs. (2.1) and (2.2) for a specified difference in angular velocities of the superfluid core and the normal crust $\Omega_s - \Omega_n$. In the case of normal quark matter core and for a specified surface current $i =$ const. the magnetic field would be uniform inside the quark core and dipolar outside the core [14, 27]. However, the quark core is in a superfluid and superconducting state, which results in a change in magnetic field distribution and the appearance of differential rotation in the strange quark star.

3. Rotation of a two-component quark star

New non-Abelian superfluid and superconducting vortices M_1 were found in Ref. [28], on the basis of a topological and group theoretical analysis of the free Ginzburg-Landau energy for the CFL phase. These vortices simultaneously exhibit quantized mechanical moment and quantized magnetic flux and their density is proportional to the angular velocity of quark matter Ω_s

$$n = \frac{2\Omega_s}{\chi}, \quad \chi = \frac{\pi\hbar}{m_B}, \tag{3.1}$$

where χ is the quantum of circulation for superfluid vortices M_1, and m_B is the baryon mass. It follows From this expression, that when the quark core slows down, i.e., when $\dot{\Omega}_s < 0$, the density of superfluid vortices M_1 is reduced, which means that these vortices move outward. Since they also possess magnetic moment, electron will by scattered by the vortex magnetic field M_1. Thus, the motions of the superfluid and normal components of the quark star are coupled because the motion of superfluid vortices M_1 is accompanied by friction with the normal component of the star. Since this is analogous to the situation that occurs in the case of rotation of superfluid in neutron stars, we can take over the corresponding equations of rotational dynamics examined in Ref. [29] and apply them in our study a two-component quark star. These equations are of the following form [29]

$$\Delta\dot{\Omega} + \dot{\Omega}_n = -\frac{\Delta\Omega}{\tau_0}, \tag{3.2}$$

$$I_s \frac{\Delta\Omega}{\tau_0} = K_{int}, \tag{3.3}$$

$$I_s \Delta\dot{\Omega} + I\dot{\Omega}_n = K_{ext}, \tag{3.4}$$

where

$$\tau_0 = \frac{1}{2k\Omega_s}, \quad k = \frac{\chi\rho_s/\eta}{1+(\chi\rho_s/\eta)^2}, \quad I = I_s + I_n, \quad \Delta\Omega = \Omega_s - \Omega_n, \tag{3.5}$$

and I_n, Ω_n and I_s, Ω_s are the moments of inertia and angular velocities of the normal and superfluid components, K_{int} is the internal moment of forces interacting between the normal and superfluid components, K_{ext} is the external retarding moment acting on the star, ρ_s is the density of the superfluid matter, while η is the friction coefficient between the vortex and the normal component. From equations (3.2)-(3.4) , we may obtain an equation that defines $\Delta\Omega$:

$$\Delta\dot{\Omega} + \frac{\Delta\Omega}{\tau_0'} = -\gamma, \tag{3.6}$$

where

$$\tau_0' = \tau_0 \frac{I_n}{I}, \quad \gamma = \frac{K_{ext}}{I_n}. \tag{3.7}$$

We assume that the star was rotating uniformly, i.e., $\Delta\Omega = 0$, at the moment when quark matter made a transition to the superfluid state. When subjected to an external retarding moment of forces, the normal component of the star continuously slows down, and consequently, $\Delta\Omega$ and K_{int} increase to the steady-state value $\Delta\Omega_{st}$. The latter is defined from the condition $\Delta\dot{\Omega} = 0$ and the equation

$$\frac{\Delta\Omega_{st}}{\Omega_n} = \frac{I_n}{I}\frac{\gamma\tau_0}{\Omega_n} = \frac{\tau_0}{\tau}, \tag{3.8}$$

where $\tau = I\Omega_n/\gamma I_n$ is the age of the quark star. As follows from expression (2.1), the surface current i of a two-component, rotating quark star that generates a magnetic field is proportional to $\Delta\Omega = \Omega_s - \Omega_n$. For generation of large magnetic fields the condition $\Delta\Omega \gg \Omega_n$ is required or $\tau_0 \gg \tau$ (according to Eq. (3.8)). This condition is satisfied by suitable selection of parameter k. For example, a value of $\tau_0 = 10^2\tau = 10^8$years ($\tau = 10^6$years is the age of a typical neutron star) may be obtained if we take $k = 10^{-18}$. From (3.5) it is easy to see that such a value for k is obtained both for large values $\eta \sim 10^{30}$ g/cm/s, and for small values $\eta \sim 10^{-6}$ g/cm/s. In the case of a strange quark star, the electron plasma is situated in a thin, near-surface layer. Consequently, electron interaction with quark vortices will result in small values for the friction coefficient η and to large values of $\Delta\Omega$. Thus all of the prerequisites exist in a strange quark star for the occurrence of differential rotation and surface currents, which result in the generation of a magnetic field of quark star.

4. Distribution of the magnetic field in the quark star

We consider a quark star of radius R, possessing a spherical core of radius a, consisting of color superconducting quark matter. The core is surrounded by a normal component consisting of an electron layer and a crust having an overall thickness equal to $R-a$. Differential rotation of the superfluid quark core and the normal component results in the generation of a magnetic field. Rotation of the quark color charge also generates a gluomagnetic field in the CFL phase of quark matter. It turns out that the electromagnetic and gluomagnetic fields are interrelated owing to the complex structure of one of the gluons, which results in so-called "rotational electromagnetism". The magnetic and gluomagnetic field may be described by vector potentials $\boldsymbol{A}(r,\vartheta)$ and $\boldsymbol{A}_8(r,\vartheta)$, which are governed by the Ginzburg-Landau equations [28, 30–33]:

$$\lambda_q^2 \mathrm{rot}\,\mathrm{rot}\,\boldsymbol{A} + \sin^2\alpha \boldsymbol{A} = \boldsymbol{f}\sin\alpha + \sin\alpha\cos\alpha\,\boldsymbol{A}_8, \tag{4.1}$$

$$\lambda_q^2 \mathrm{rot}\,\mathrm{rot}\,\boldsymbol{A}_8 + \cos^2\alpha\,\boldsymbol{A}_8 = -\boldsymbol{f}\cos\alpha + \sin\alpha\cos\alpha\,\boldsymbol{A}, \tag{4.2}$$

where the penetration depth λ_q and the angle of magnetic and gluomagnetic field "mixing" are defined in Refs. [33, 34]. Since quark matter in the CFL phase is a type-II superconductor, the magnetic and gluomagnetic fields may penetrate into the quark core by means of these quantum vortices. To find the mean magnetic field in the quark core, the system of equations (4.1) and (4.2) must be solved for $\boldsymbol{A}(r,\vartheta)$ and $\boldsymbol{A}_8(r,\vartheta)$. The corresponding solution is of the form [34]:

$$A_\varphi(r,\vartheta) = \left(M_\varphi(r) + \mathrm{ctg}^2\alpha M_\varphi(r)\frac{r}{a} + \frac{c_0 r}{\sin\alpha}\right)\sin\vartheta = A_\varphi(r)\sin\vartheta, \tag{4.3}$$

$$A_{8\varphi}(r,\vartheta) = -\left(M_\varphi(r) - \frac{r}{a}M_\varphi(a)\right)\mathrm{ctg}\alpha\sin\vartheta = A_{8\varphi}(r)\sin\vartheta, \tag{4.4}$$

where $M_\varphi(r)$ is defined by the following expression:

$$M_\varphi(r) = \frac{c_1}{r^2}\left[\mathrm{sh}\frac{r}{\lambda_q} - \frac{r}{\lambda_q}\mathrm{ch}\frac{r}{\lambda_q}\right]. \tag{4.5}$$

In expressions (4.3) and (4.5) c_0 and c_1 are integration constants. We note that expression (4.4) for A_8 satisfies the gluon confinement condition, i.e., the disappearance of the gluomagnetic field at the boundary of the quark core: $A_8(a,\theta) = 0$, while $M_\varphi(r)$ satisfies the condition $\lim_{r\to 0} M_\varphi(r) = 0$. The magnetic field components in the quark core $\boldsymbol{B}^q$ are linked to the component of vector potential A_φ via the familiar relations:

$$B_r^q = \frac{1}{r\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta A_\varphi(r,\vartheta)\right), \quad B_\vartheta^q = -\frac{1}{r}\frac{\partial}{\partial r}\left(rA_\varphi(r,\vartheta)\right). \tag{4.6}$$

Inserting the first of the solutions (4.5) in definition (4.6), we finally obtain the magnetic field components in the quark phase, i.e., when $r \le a$:

$$B_r^q = \left[\frac{2M_\varphi(r)}{r} + 2ctg^2\alpha\frac{M_\varphi(a)}{a} + \frac{2c_0}{\sin\alpha}\right]\cos\vartheta,$$

$$B_\vartheta^q = -\left[\frac{1}{r}\frac{\partial}{\partial r}\left(rM_\varphi(r)\right) + 2ctg^2\alpha\frac{M_\varphi(a)}{a} + \frac{2c_0}{\sin\alpha}\right]\sin\vartheta. \tag{4.7}$$

Now we consider the normal electron layer in the region $a < r < a + z_e$. There, the vector potential $\boldsymbol{A}^n$ is determined from the equation $\mathrm{rot}\,\mathrm{rot}\boldsymbol{A}^n = 0$, the solution of which is of the form:

$$A_\varphi^n(r,\theta) = c_2 r\sin\theta = A_\varphi^n(r)\sin\theta, \tag{4.8}$$

where, c_2 is an integration constant. The magnetic field in the electron layer $a < r < a + z_e$ was found in Ref. [33, 34] and has the form

$$B_r^e = \left[\frac{2A_\varphi^n(r)}{r} + B\right]\cos\vartheta, \quad B_\vartheta^e = -\left[\frac{1}{r}\frac{\partial}{\partial r}\left(rA_\varphi^n(r)\right) + B\right]\sin\vartheta, \tag{4.9}$$

where B will characterize the mean value of the magnetic field in this layer. In the quark star crust, i.e., in the region $a + z_e < r < R$, the magnetic field is dipolar and is given by

$$B_r^c = \frac{2\mathfrak{M}}{r^3}\cos\vartheta, \quad B_\vartheta^c = -\frac{\mathfrak{M}}{r^3}\sin\vartheta, \tag{4.10}$$

where $\mathfrak{M}$ is the total magnetic moment of the rotating quark core. The constants that enter into the solution (4.7), (4.9), and (4.10) are determined by the requirement for continuity of magnetic field components at the surface of the quark core, where $r = a : B_r^q(a) = B_r^e(a)$ and $B_\vartheta^q(a) = B_\vartheta^e(a)$. These conditions may be expressed as:

$$\frac{2M_\varphi(a)}{a} + 2\mathrm{ctg}^2\alpha\frac{M_\varphi(a)}{a} + \frac{2c_0}{\sin\alpha} = \frac{2A_\varphi^n(a)}{a} + B,$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rM_\varphi(r)\right)\Big|_{r=a} + 2\mathrm{ctg}^2\alpha\frac{M_\varphi(a)}{a} + \frac{2c_0}{\sin\alpha} = \frac{1}{r}\frac{\partial}{\partial r}\left(rA_\varphi^n(r)\right)\Big|_{r=a} + B. \tag{4.11}$$

From these equations one finds [17]

$$c_1 = 0, \quad c_2 = -\frac{D}{a^3\sin^2\alpha}, \quad D = \frac{Ba^3}{2}\sin^2\alpha - c_0 a^3\sin\alpha. \tag{4.12}$$

We shall also take advantage of the continuity of the magnetic field normal component in the transition from the electron layer to the star core:

$$B_r^e = B_r^c, \quad \Rightarrow \quad \frac{2A_\varphi^n(r)}{r} + B = \frac{2\mathfrak{M}}{r^3},$$

or

$$2c_2 + B = \frac{2\mathfrak{M}}{a^3} = b - \frac{2D}{a^3 \sin^2 \alpha}. \tag{4.13}$$

Thus, taking into account Eqs. (4.12) and (4.13), the magnetic field in the quark core, i.e. in the region $r \leq a$, is given by

$$B_r^q = \frac{2c_0}{\sin\alpha} \cos\vartheta = \frac{2\mathfrak{M}}{a^3} \cos\vartheta, \quad B_\vartheta^q = -\frac{2c_0}{\sin\alpha} \sin\vartheta = -\frac{2\mathfrak{M}}{a^3} \sin\vartheta. \tag{4.14}$$

The field in the normal electron layer $a \leq r \leq a + z_e$ is given by

$$B_r^n = (2c_2 + B)\cos\vartheta = B_r^q, \quad B_\vartheta^n = -(2c_2 + B)\sin\vartheta = B_\vartheta^q. \tag{4.15}$$

And finally, the field outside the core is determined using relations (4.10), and as follows from these relations, the value of the external magnetic field at the pole and at the equator of the star are equal to

$$B_p^{ext} = \frac{2\mathfrak{M}}{R^3} = B^q \left(\frac{a}{R}\right)^3, \quad B_e^{ext} = \frac{\mathfrak{M}}{R^3} = \frac{B^q}{2} \left(\frac{a}{R}\right)^3, \tag{4.16}$$

Thus, the mean magnetic field of the quark core and of the magnetic field at the star's surface is completely determined by specifying the total magnetic moment $\mathfrak{M}$ of the core, which is equal to $B^q a^3/2$.

We proceed to a calculation of the total magnetic moment $\vec{\mathfrak{M}}$ of a strange quark star. First, we note that the magnetic moment is directed along the star's rotation axis; consequently, only the z component of vector $\vec{\mathfrak{M}}$, which we have previously designated as $\mathfrak{M}$, is non-zero. The quantity $\mathfrak{M}$ may be determined from the boundary condition of the tangential component of the magnetic field at the core surface. Indeed, this condition is

$$B_\vartheta^c(a) - B_\vartheta^n(a) = \frac{4\pi}{c} i', \tag{4.17}$$

where i' is expressed using the surface current density i, specified by relation (2.1), as follows

$$i' = i 2\pi a \sin\vartheta = \sigma (\Omega_s - \Omega_n) a \sin\vartheta. \tag{4.18}$$

Inserting the values of the ϑ components of magnetic fields from (4.15) and (4.10) into expression (4.17), we obtain a relation that determines $\mathfrak{M}$

$$\mathfrak{M} = \frac{4\pi\sigma}{3c} a^4 (\Omega_s - \Omega_n). \tag{4.19}$$

Here, $4\pi\sigma = E$ is the electric field created in the double charged layer at the star's surface. Taking account of this, relation (4.19) assumes the form

$$\mathfrak{M} = \frac{Ea^4}{3c} \Delta\Omega. \tag{4.20}$$

For the magnetic field on star's surface we obtain:

$$B^{ext} \sim \frac{\mathfrak{M}}{R^3} = \frac{Ea^4}{3cR^3} \Delta\Omega \tag{4.21}$$

Thus, the star's magnetic moment $\mathfrak{M}$ and external magnetic field B are proportional to the difference between the angular velocities of the quark matter and the electron plasma $\Delta\Omega$. When the star's rotation slows down this difference increases, since Ω_n initially decreases and then only Ω_n follows its change. The increase in $\Delta\Omega$ leads to a rise in the tangential component of the magnetic field in the normal phase. When this field reaches the value of H_{c1} characterizing the lower superconducting critical field for quark matter, the vortices formed at the surface of the quark core move inward carrying magnetic flux with them. Assuming that the characteristic time for redistribution of the vortices inside the quark core is shorter than the characteristic time for the increase in the magnetic field in the normal phase, the field B^q can follow the rise in the field B^n. Therefore, the steady-state condition $B^q = B^n$ is satisfied almost all the time. We now prove that this proposition is true.

In fact, according to Refs. [35, 36], the characteristic time for redistribution of M_1 vortex filaments with magnetic flux is given by

$$\tau_1 = \frac{1}{k}\frac{mc}{e\bar{B}} = \frac{1}{k\Omega_L}, \tag{4.22}$$

where Ω_L is the Larmor frequency for the quark matter and $\bar{B}$ is the average magnetic induction. Here the constant k is given by Eq. (3.5). The characteristic rise time was found in Section 3 and is given by (cf. Eqs. (3.5) and (3.6))

$$\tau_0' = \frac{1}{2k\Omega_s}\frac{I_n}{I}. \tag{4.23}$$

From Eqs. (4.22) and (4.23) we have

$$\frac{\tau_1}{\tau_0'} = \frac{2\Omega_s}{\Omega_L}\frac{I}{I_n}. \tag{4.24}$$

We now estimate this ratio. Taking $\Omega_s \approx 700$ Hz, $\Omega_L \approx 10^{18}$ Hz and $I/I_n \approx 10^{11}$, we obtain for the ratio τ_1/τ_0' a value on the order of 10^{-5}, which confirms the above proposition.

5. Models of strange quark stars and their magnetic fields

Strange quark matter is a possible state of cold matter which could be self-bound by the strong interaction. Strange quark matter can be described using three phenomenological parameters: the MIT bag model constant B, the quarkâĂŞgluon interaction constant α_c, and the mass of the strange quark m_s. The existence of self-bound strange stars is possible for certain values of these parameters. The principal properties of stellar configurations consisting of strange quark core and a crust were studied in Refs. [13, 22]. We will use the result of Ref. [13] since it contains detailed information needed for our estimates in this Section. Table 1 shows the integral parameters for static strange stars with a crust for various central densities and for $B = 60$ MeV/fm^3 [13]. As seen from Table 1, for small stellar masses, the crust thickness attains a value of the order of 35% of the overall star radius, while for large star masses, it is of the order of 5%. The parameters of strange dwarfs with masses $M < 0.02M_\odot$ will be discussed separately at the end of this Section. We note that rotation leads to both an increase in the mass of the configuration and to an increase in its radius. However most known pulsars rotate with angular velocities which are much smaller than the Keplerian angular velocities. For this reason, we will hereafter use the integral parameters of static configurations in order to estimate the magnitude of the magnetic field at the surface of a strange quark star. Finally, we evaluate the value of the magnetic field B of quark star models using the

$M/M_\odot$	R, km	ΔR_{cr}, km
0.1	6	1.8
0.5	8	0.8
1	9.2	0.5

Table 1: Integral parameters of quark stars. Here M is the star mass in units of the solar mass ($M_\odot$), R is the radius of the star and ΔR_{cr} is the thickness of the crust.

formula (4.21). The maximum magnetic field may be obtained if we assume that $\Delta\Omega \sim \Omega_s \sim 10^4$ rad/s, which is the maximum possible value for bare quark star [37]. For the electric field we take the value $E \sim 10^{18}$ V/cm $\sim 3 \cdot 10^{15}$ cgs units. If for the model of a quark star with mass $M/M_\odot = 1$ we take the radius of a neutron star $a \sim R \sim 10$ km, then we obtain magnetic field at the star's surface $B^{ext} \sim 3 \cdot 10^{14}$ G, which is of the order of a hypothesized magnetar field.

Besides magnetars, i.e., objects with superpowerful magnetic fields, objects have also been observed with small magnetic field values, of the order of $10^{10} - 10^{11}$ G, the so-called central compact objects (CCO) [38]. Such magnetic field values may also be explained in terms of the quark star model. It follows from Table 1 that a magnetic field value of the order of 10^{11} G can be obtain for a quark star model having a mass of the order of $M/M_\odot = 0.5$, electric field $E_{18} \sim 10^{17}$ V/cm and the difference of angular velocities $\Delta\Omega \sim 10^2$ rad/s.

We can provide yet another argument in favor of the quark model of magnetars. If the observable magnetic fields at their surfaces attain the order of 10^{15} G, then the interior field may exceed the surface value by an order of magnitude. Ref. [39] examined the possibility of destruction of proton superconductivity in the interior regions of neutron stars, where the magnetic field exceeds the value of the upper critical field for the proton superconductor. Their calculations show that in the larger part of the hadronic matter region, especially for dense configurations, protons make a transition to the normal state, which would result in the destruction of the proton vortex structure. However, in the quark star model of magnetars vortex structures are preserved, since the value of the upper critical field for quark matter is much higher than the one that was considered in Ref. [39]. As shown in Ref. [40], the motion of vortex lattices results in the observation of sudden changes in the angular rotation velocities of compact stars. Observing sudden changes in the angular velocity of magnetars would support the magnetar model considered by us. To date, one such sudden change has been observed in anomalous X-ray pulsar 1RXS J170849.0-400910 [41].

We have also calculated the values of magnetic fields for models of strange dwarfs given in Ref. [22]. The results of these calculations are shown in Table 2. As is seen from Table 2, the maximum values of magnetic field can attain $B \sim 10^3 - 10^5$ G. In Ref. [20] seven objects were chosen among white dwarfs as candidates for strange dwarfs. Magnetic fields of five of these seven were given in Refs. [42–44]. The object names, masses and observed values of magnetic fields of these five stars are shown in Table 3. As is seen from calculated values of magnetic field in Table 2 the mechanism of generation of magnetic field discussed in this work can explain the observed

$\rho_{\rm crust}$, g/cm^3	$M_{\rm core}/M_\odot$	$M/M_\odot$	R_{core}, km	R, km	$B_{\max}$, kG
$4.3\cdot 10^{11}$	0.01405	0.9646	2.561	2347	110
$4.3\cdot 10^{11}$	0.01732	0.7232	2.745	5280.7	13
10^{10}	0.00133	1.0145	1.170	2289.9	5
10^{10}	0.00303	0.7938	1.538	5136.1	1.4
10^{9}	0.00080	0.7625	1.005	5519.5	0.2

Table 2: Integral parameters of strange dwarfs. $\rho_{\rm crust}$ is crust density, M and $M_{\rm core}$ are masses of star and quark core, respectively; R and $R_{\rm core}$ are radii of the star and the quark core, respectively, [22], $B_{\max}$ is the maximum value of magnetic field on the surface of the star.

Star number	Star name	$M/M_\odot$	B, kG
WD 1134+300	GD 140	0.79	0.6
WD 0644+375	EG 50	0.5	5
WD 0148+467	GD 279	0.44	6
WD 2007-303	LTT 7987	0.44	<10
WD 1337+705	G 238-44	0.42	17

Table 3: Observed parameters of compact objects, which were proposed in Ref. [20] as candidates for strange dwarfs. Here M is the mass of the star, B is average magnetic field on the star surface.

values of magnetic fields listed in Table 3. However, larger values of magnetic fields $B \geq 10^5 G$ were reported recently in Ref. [45]. The present mechanism of magnetic field generation cannot explain magnetic fields of white dwarfs of such magnitude.

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