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THERMOELASTIC STRIP-SHAPED PLATE MOTION CONTROL: OPTIMAL RESTORATION OF DEFLECTION FROM INCOMPLETE MEASUREMENTS WITH ERRORS

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Abstract. We consider a problem of motion control for a thermoelastic strip-shaped plate, namely, the problem of optimal restoration of its deflection in the presence of errors in temperature measurements. By the separation of variables, this problem is reduced to a problem for an infinite system of ordinary differential equations involving real signal observation. For each harmonic, using the incoming signal boost, we construct a universal optimal operation that allows us to restore the plate's deflection at any of its points and at any time instant.

Introduction

The theory of optimal control of systems with distributed parameters and their observation has an ever increasing sphere of applications. For optimal control of dynamical systems it is necessary to know the current state of the process, which can hardly be accomplished with precision on the basis of direct measurements, since, firstly, not all of the variables can be measured and, secondly, the measurements, apart from the desired signal, contain noise. In such situations, the characteristics of motion of a system have to be restored with the greatest possible precision.

Restoration of the unknown characteristics and control of dynamical systems with distributed parameters is discussed in [1, 2], where an extensive bibliography is given. In [3–5], the state of all points of a system at any time instant is restored by the method of separation of variables on the basis of some data from the history of the signal.

Here, we consider a thermoelastic strip-shaped plate (coupled problem) subject to distributed control [6]. The problem is to restore its deflection on the basis of actual (with errors) signals from measuring devices. This work is related to [1–5].

1. Setting of the Problem

Consider the problem of transverse vibrations of a plate coupled with the heat transfer problem, which we write in classical form (see [6–8]) as follows:

$$D\left[\frac{\partial^4 w}{\partial t^4} + \alpha(1+\nu)\frac{\partial^2 T}{\partial t^2}\right] + \rho_0 h\frac{\partial^2 w}{\partial t^2} = F(x,t), \tag{1.1}$$

$$\frac{\partial T}{\partial t} - \chi\frac{\partial^2 T}{\partial x^2} + \left(\frac{12\chi}{h^2} + \frac{6k^*}{\rho_0 h^2 c_p}\right)T - \chi\eta\frac{\partial^3 w}{\partial x^2 \partial t} = \frac{12Qz_0}{\rho_0 h^3 c_p} + \frac{6k^*(T_+ - T_-)}{\rho_0 h^2 c_p} = \Phi(x,t). \tag{1.2}$$

Here, we use the common notation borrowed from [1–3], and therefore, omit the explanations.

Consider the boundary conditions

$$w = M = -D\left[\frac{\partial^2 w}{\partial x^2} + \alpha(1+\nu)T\right] = 0, \quad T = \Phi_1(t) \text{ for } x = 0,$$
$$w = M = 0, \quad T = \Phi_2(t) \text{ for } x = l. \tag{1.3}$$

The functions $F(x,t)$ and $\Phi(x,t)$ describe the control actions.

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In order to synthesize the control actions (or optimal actions) on the basis of feedback, one needs the complete data about the state of the systems, i.e., the values of $w(x,t)$ at each time instant t and at each point $x \in [0,l]$.

Suppose that the measuring devices allow us to measure some quantity defined on a time interval $t-\vartheta \le \tau \le t$, where $\vartheta > 0$ is a constant equal to the length of the time interval on which some history of the incoming signal is taken into account.

In contrast to the incoming signal, known within the limits of the interval $t-\vartheta \le \tau \le t$, the values of the control actions $F(x,\tau)$ and $\Phi(x,\tau)$ at the instant t are not included into the history taken into account. For the functions $F(x,\tau)$ and $\Phi(x,\tau)$, the argument τ varies within the semi-interval $t-\vartheta \le \tau < t$, since the deflection value $w(x,t)$ at the instant t is needed for determining the controls at this instant.

In real situations, the observed (measured) variables are usually some functionals determined by the systems's state. It is impossible, in principle, to measure precisely the state of a system at any point of $[0,l]$. Thus, apart from the desired signal, the observed variables may contain measurement errors.

Assume that the observed actual signal $Z(\tau)$ is related to the temperature of the plate $T(x,\tau)$ and the measurement error $S\big(x,\xi(\tau)\big)$, which is a random process, by the equation (see [5])

$$Z(\tau) = \int_0^l N(x,\tau)\big[T(x,\tau) + S\big(x,\xi(\tau)\big)\big]\,dx, \quad t-\vartheta \le \tau \le t. \tag{1.4}$$

The function $N(x,\tau)$ is known and characterizes the measurement method and the parts of the object to be measured. This function may be stationary, i.e., of the form $N(x)$.

The set $X_N = \{x\colon x \in [0,l],\ N(x,\tau) \ne 0\}$ is the base of the measuring device. It may consist of isolated points of the interval $[0,l]$, be a subset of $[0,l]$ (a union of several subsets), or coincide with $[0,l]$. Suppose, for instance, that we are measuring temperature at a fixed point $x_0 \in [0,l]$. Then

$$Z(\tau) = T(x_0,\tau) + S\big(x_0,\xi(\tau)\big) = \int_0^l \delta(x-x_0)\big[T(x,\tau) + S\big(x,\xi(\tau)\big)\big]\,dx.$$

Here, $N(x,\tau) = \delta(x-x_0)$, and the base of the measuring device is the single point $x = x_0$.

Our aim is the following: given the observed signal $Z(\tau)$, $\tau \in [t-\vartheta, t]$, restore the deflection $w(x,t)$ of the plate at any instant t and any point $x \in [0,l]$, no matter what value of $w(x,t)$ has been realized in the process (1.1), (1.2).

2. Reduction of the Original Problem to the Problem of Observation of Systems Described by Ordinary Differential Equations

Taking into account the boundary conditions (1.3) , we represent the solution of equations (1.1), (1.2) in the form

$$w(x,t) = \sum_{k=1}^{\infty} w_k(t)\sin\lambda_k x, \quad \text{where} \quad w_k(t) = \frac{2}{l}\int_0^l w(x,t)\sin\lambda_k x\,dx, \tag{2.1}$$

$$T(x,t) = \sum_{k=1}^{\infty} T_k(t)\sin\lambda_k x,$$

where

$$T_k(t) = \frac{2}{l}\int_0^l T(x,t)\sin\lambda_k x\,dx.$$

Assuming that the functions $F(x,t)$ and $\Phi(x,t)$ can also be represented as

$$F(x,t)=\sum_{k=1}^{\infty} f_k(t)\sin\lambda_k x,\quad f_k(t)=\frac{2}{l}\int_0^l F(x,t)\sin\lambda_k x\,dx,$$

$$\Phi(x,t)=\sum_{k=1}^{\infty} g_k(t)\sin\lambda_k x,\quad g_k(t)=\frac{2}{l}\int_0^l \Phi(x,t)\sin\lambda_k x\,dx,$$

from (1.1), (1.2), (2.1) , we obtain the following infinite systems of differential equations

$$\begin{gathered}\frac{d^2w_k}{dt^2}+\omega_k^2 w_k-a_kT_k=f_k(t),\\ \frac{dT_k}{dt}+\chi_kT_k+b_k\frac{dw_k}{dt}=\psi_k(t),\quad k=1,2,\dots,\end{gathered}\tag{2.2}$$

where

$$\omega_k=\frac{D}{\rho_0 h}\lambda_k^4,\quad a_k=\frac{\alpha(1+\nu)}{\rho_0 h}\lambda_k^2,\quad b_k=\chi\eta\lambda_k^2,\quad \lambda_k=\frac{\pi k}{l},$$

$$\chi_k=\frac{12\chi}{h^2}+\frac{6k^*}{\rho_0h^2c_p}+\chi\lambda_k^2,\quad \psi_k=\chi\lambda_k[\Phi_1(t)-(-1)^k\Phi_2(t)]+g_k.$$

For each harmonic, let

$$x_{1k}=\omega_k w_k,\quad x_{2k}=\dot w_k,\quad x_{3k}=T_k,\quad u_{1k}=f_k,\quad u_{2k}=\psi_k.$$

Then, (2.2) becomes

$$\dot x_{1k}=\omega_k x_{2k},\quad \dot x_{2k}=-\omega_k x_{1k}+a_kx_{3k}+u_{1k},\quad \dot x_{3k}=-\chi_kx_{3k}-b_kx_{2k}+u_{2k},\tag{2.3}$$

which can be written in terms of matrices as

$$\dot x_k=A_kx_k+B_ku_k,$$

where

$$A_k=\begin{pmatrix}0&\omega_k&0\\-\omega_k&0&a_k\\0&-b_k&-\chi_k\end{pmatrix},\quad B_k=\begin{pmatrix}0&0\\1&0\\0&1\end{pmatrix},\quad x_k=\begin{pmatrix}x_{1k}\\x_{2k}\\x_{3k}\end{pmatrix},\quad u_k=\begin{pmatrix}u_{1k}\\u_{2k}\end{pmatrix}.$$

Assuming that the measurement error $S\big(x,\xi(\tau)\big)$ can be expanded as

$$S\big(x,\xi(\tau)\big)=\sum_{k=1}^{\infty}s_k(\tau)\sin\lambda_kx,\qquad s_k(\tau)=\frac{2}{l}\int_0^l\big(x,\xi(\tau)\big)\sin\lambda_kx\,dx$$

and using (2.1), (1.4), we get

$$Z(\tau)=\int_0^l N(x,\tau)\Big[\sum_{k=1}^{\infty}T_k(\tau)\sin\lambda_kx+\sum_{k=1}^{\infty}s_k(\tau)\sin\lambda_kx\Big]\,dx=\sum_{k=1}^{\infty}[N_k(\tau)T_k(\tau)+\Delta_k(\tau)],$$

where

$$N_k(\tau)=\int_0^l N(x,\tau)\sin\lambda_kx\,dx,\quad \Delta_k(\tau)=N_k(\tau)s_k(\tau).$$

For a stationary $N(x)$, the coefficients N_k are constants.

Thus, for each harmonic (2.2), the actual incoming signal can be represented as follows:

$$Z_k(\tau)=N_k(\tau)T_k(\tau)+\Delta_k(\tau),\quad \tau\in[t-\vartheta,t].$$

The error $\Delta_k(\tau)$ is unknown (since $s_k(\tau)$ is unknown), but it can be assumed that the physical setting of the measurement process implies some estimate of this error. Suppose that $\Delta_k(\tau)$ is an element of L_2. Then, the estimate for the possible measurement error $\Delta_k(\tau)$ can be written in the form

$$\rho[\Delta_k(\cdot)] = \left(\int\limits_{t-\vartheta}^{t} \Delta_k^2(\tau)\, d\tau \right)^{1/2} \leq \delta_k, \tag{2.4}$$

where δ_k $(k = 1, 2, \ldots)$ are positive constants. If there is no measurement error, then the measurement is precise (and the incoming signal is ideal), but incomplete.

It is required to find an operation $\varphi_k^0[t, z_k(\tau)]$ that satisfies the condition

$$\sup_{z_k} |\varphi_k^0[t, z_k(\tau)] - w_k(\tau)| = \min_{\varphi_k} \sup_{z_k} |\varphi_k[t, z_k(\tau)] - w_k(\tau)| \tag{2.5}$$

over all realizations $z_k(\tau)$ and all operations φ_k.

Note that the operation φ_k on which the supremum in (2.5) is realized for any t satisfies the condition (see [9, 10])

$$\varphi_k[t, N_k(\tau)T_k(\tau)] = w_k(t). \tag{2.6}$$

From (2.6) and the linear structure of φ_k, we obtain

$$\varphi_k[t, z_k(\tau)] - w_k(t) = \varphi_k[t, \Delta_k(\tau)].$$

Since (2.4) ensures that

$$\sup_{\Delta_k} |\varphi_k[t, \Delta_k(\tau)]| = \delta_k \rho^*[\varphi_k], \tag{2.7}$$

it follows that in order to solve our problem, we have to find an optimal operation

$$\varphi_k^0[t, N_k(\tau)T_k(\tau)] = w_k(t)$$

with the smallest possible norm $\rho^*[\varphi_k^0]$ for each t. Therefore, we have to construct the resolving operation for the ideal signal

$$N_k(\tau)T_k(\tau), \quad \tau \in [t - \vartheta, t]. \tag{2.8}$$

Given the signal $\{N_k(\tau)T_k(\tau), f_k(\tau), \psi_k(\tau)\}$, where $\tau \in [t - \vartheta, t]$ for $N_k(\tau)T_k(\tau)$ and $\tau \in [t - \vartheta, t)$ for $f_k(\tau)$ and $\psi_k(\tau)$, we have to construct a linear operation such that

$$\varphi_k[t, \{N_k(\tau)T_k(\tau), f_k(\tau), \psi_k(\tau)\}] = w_k(t), \quad k = 1, 2, \ldots \tag{2.9}$$

For each $k = 1, 2, \ldots$, consider an "amplified signal" (with respect to the signal (2.8); see [4, 5])

$$y_k(\tau) = \lambda_k^\varepsilon N_k x_{3k}(\tau), \quad \tau \in [t - \vartheta, t], \tag{2.10}$$

where $T_k(\tau) = x_{3k}$, $\varepsilon > 1$ is a constant. It is not very difficult to realize this signal.

Thus, for each $k = 1, 2, \ldots$, we have an observation problem for system (2.3) with signal (2.10).

For system (2.3) to be completely observable with signal (2.10), in the case of $N_k(\tau) = \text{const}$, for each $k = 1, 2, \ldots$, it is necessary and sufficient that that rank of the matrix

$$\begin{Bmatrix} G_k' & A_k' G_k' & A_k'^2 G_k' \end{Bmatrix}$$

be equal to 3 (see [9]), where

$$G_k = \begin{pmatrix} 0 & 0 & \lambda_k^\varepsilon N_k \end{pmatrix}.$$

Here and in what follows, the prime indicates the transpose of a matrix.

Note that the conditions of complete observability are satisfied, since $b_k \neq 0$, $\omega_k \neq 0$, $N_k \neq 0$.

3. Solving the Problem

The resolving operation $\varphi_k[t, \{y_k(\tau), u_k(\tau)\}]$ is constructed as follows [9, 10]:

$$\varphi_k[t, \{y_k(\tau), u_k(\tau)\}] = \varphi_k^0[t, y_k(\tau)] - \varphi_k^0\left[t, G_k \int\limits_t^\zeta H_k[\zeta, \tau] u_k(\tau)\, d\tau\right], \tag{3.1}$$

where φ_k^0 is the resolving operation with $u_k \equiv 0$, i.e.,

$$\varphi_k^0[t, y_k(\tau)] = w_k(t), \tag{3.2}$$

and $H_k[\zeta, \tau] = X_k[\zeta, \tau]B_k$, $X_k[\zeta, \tau]$ is the normalized fundamental matrix for the homogeneous problem for system (2.3),

$$X_k[\tau, t] = \begin{pmatrix} x_{k11}(\tau, t) & x_{k12}(\tau, t) & x_{k13}(\tau, t) \\ x_{k21}(\tau, t) & x_{k22}(\tau, t) & x_{k23}(\tau, t) \\ x_{k31}(\tau, t) & x_{k32}(\tau, t) & x_{k33}(\tau, t) \end{pmatrix}.$$

We will need the expressions of only three elements of this matrix, namely,

$$\begin{aligned} x_{k31}(\tau, t) &= \sigma_{k1}\delta_{k1}e^{\mu_{k1}(\tau - t)} - \sigma_{k2}\delta_{k2}e^{\mu_{k2}(\tau - t)} + \sigma_{k3}\delta_{k3}e^{\mu_{k3}(\tau - t)}, \\ x_{k32}(\tau, t) &= -\sigma_{k1}\beta_{k1}e^{\mu_{k1}(\tau - t)} + \sigma_{k2}\beta_{k2}e^{\mu_{k2}(\tau - t)} - \sigma_{k3}\beta_{k3}e^{\mu_{k3}(\tau - t)}, \\ x_{k33}(\tau, t) &= \sigma_{k1}e^{\mu_{k1}(\tau - t)} - \sigma_{k2}e^{\mu_{k2}(\tau - t)} + \sigma_{k3}e^{\mu_{k3}(\tau - t)}, \end{aligned} \tag{3.3}$$

where

$$\begin{gathered} \sigma_{k1} = \frac{\mu_{k1}^2 + \omega_k^2}{(\mu_{k2} - \mu_{k1})(\mu_{k3} - \mu_{k1})}, \quad \sigma_{k2} = \frac{\mu_{k2}^2 + \omega_k^2}{(\mu_{k2} - \mu_{k1})(\mu_{k3} - \mu_{k2})}, \quad \sigma_{k3} = \frac{\mu_{k3}^2 + \omega_k^2}{(\mu_{k3} - \mu_{k1})(\mu_{k3} - \mu_{k2})}, \\ \delta_{k1} = \frac{\mu_{k2}\mu_{k3} - \omega_k^2}{a_k\omega_k}, \quad \delta_{k2} = \frac{\mu_{k1}\mu_{k3} - \omega_k^2}{a_k\omega_k}, \quad \delta_{k3} = \frac{\mu_{k1}\mu_{k2} - \omega_k^2}{a_k\omega_k}, \\ \beta_{k1} = \frac{\mu_{k3} + \mu_{k2}}{a_k}, \quad \beta_{k2} = \frac{\mu_{k3} + \mu_{k1}}{a_k}, \quad \beta_{k3} = \frac{\mu_{k2} + \mu_{k1}}{a_k}. \end{gathered} \tag{3.4}$$

Here, μ_{k1}, μ_{k2}, μ_{k3} are the roots of the characteristic equation for the homogenous part of system (2.3):

$$\mu_k^3 + \chi_k\mu_k^2 + (\omega_k^2 + a_k b_k)\mu_k + \chi_k\omega_k^2 = 0,$$

and these roots are assumed mutually distinct.

The solution of system (2.3), under the conditions that $u_k = 0$ for each $k = 1, 2, \ldots$, can be written in the form $x_k(\tau) = X_k[\tau, t]x_k(t)$. Therefore, (2.10) can be represented as

$$y_k(\tau) = \lambda_k^\varepsilon N_k[x_{k31}(\tau, t)x_{1k}(t) + x_{k32}(\tau, t)x_{2k}(t) + x_{k33}(\tau, t)x_{3k}(t)], \quad t - \vartheta \le \tau \le t. \tag{3.5}$$

The operation that calculates the function $w_k(t)$ from the signal (3.5) is sought in the form

$$\int\limits_{t-\vartheta}^{t} \bar{V}_k(t, \tau)y_k(\tau)\, d\tau = w_k(t) = \frac{1}{\omega_k}x_{1k}(t). \tag{3.6}$$

Substituting $y_k(\tau)$ from (3.5) into (3.6), changing the variable: $\tau - t = \zeta$, and setting $\bar{V}_k(t, t + \zeta) = V_k(\zeta)$, and$x_{k3i}(t + \zeta, t) = x_{k3i}(\zeta)$, $i = 1, 2, 3$, we get

$$\int\limits_{-\vartheta}^{0} x_{k31}(\zeta)V_k(\zeta)\, d\zeta = \frac{1}{\lambda_k^\varepsilon N_k\omega_k}, \quad \int\limits_{-\vartheta}^{0} x_{k32}(\zeta)V_k(\zeta)\, d\zeta = 0, \quad \int\limits_{-\vartheta}^{0} x_{k33}(\zeta)V_k(\zeta)\, d\zeta = 0. \tag{3.7}$$

For each $k = 1, 2, \ldots$, we find a function $V_k(\zeta)$ that satisfies the integral conditions (3.7) and is optimal in the following sense:

$$\int_{-\vartheta}^{0} V_k^2(\zeta)\, d\zeta \to \min\,. \tag{3.8}$$

Treating the variational problem (3.7), (3.8) as a problem for moments [9], we find its solution in the form

$$\begin{aligned} V_k^0(\zeta) = [&\sigma_{k1}(\delta_{k1} + \beta_{k1}\Delta_{k2} + \Delta_{k3})e^{\mu_{k1}\zeta} - \sigma_{k2}(\delta_{k2} + \beta_{k2}\Delta_{k2} + \Delta_{k3})e^{\mu_{k2}\zeta} \\ &+ \sigma_{k3}(\delta_{k3} + \beta_{k3}\Delta_{k2} + \Delta_{k3})e^{\mu_{k3}\zeta}] \cdot [\lambda_k^{\varepsilon} N_k \omega_k (A_{k1}\Delta_{k1} - A_{k12}\Delta_{k2} + A_{k13}\Delta_{k3})]^{-1}, \end{aligned} \tag{3.9}$$

where

$$\Delta_{k1} = A_{k2}A_{k3} - A_{k23}^2, \quad \Delta_{k2} = A_{k3}A_{k12} - A_{k13}A_{k23}, \quad \Delta_{k3} = A_{k12}A_{k23} - A_{k2}A_{k13},$$

$$A_{k1} = b_{k1}\sigma_{k1}^2\delta_{k1}^2 + b_{k2}\sigma_{k2}^2\delta_{k2}^2 + b_{k3}\sigma_{k3}^2\delta_{k3}^2 - 2\sigma_{k1}\sigma_{k2}\delta_{k1}\delta_{k2}b_{k12} + 2\sigma_{k1}\sigma_{k3}\delta_{k1}\delta_{k3}b_{k13} - 2\sigma_{k2}\sigma_{k3}\delta_{k2}\delta_{k3}b_{k23},$$

$$A_{k2} = b_{k1}\sigma_{k1}^2\beta_{k1}^2 + b_{k2}\sigma_{k2}^2\beta_{k2}^2 + b_{k3}\sigma_{k3}^2\beta_{k3}^2 - 2\sigma_{k1}\sigma_{k2}\beta_{k1}\beta_{k2}b_{k12} + 2\sigma_{k1}\sigma_{k3}\beta_{k1}\beta_{k3}b_{k13} - 2\sigma_{k2}\sigma_{k3}\beta_{k2}\beta_{k3}b_{k23},$$

$$A_{k3} = b_{k1}\sigma_{k1}^2 + b_{k2}\sigma_{k2}^2 + b_{k3}\sigma_{k3}^2 - 2\sigma_{k1}\sigma_{k2}b_{k12} + 2\sigma_{k1}\sigma_{k3}b_{k13} - 2\sigma_{k2}\sigma_{k3}b_{k23},$$

$$\begin{aligned} A_{k12} = &-2b_{k1}\sigma_{k1}^2\delta_{k1}\beta_{k1} - 2b_{k2}\sigma_{k2}^2\delta_{k2}\beta_{k2} - 2b_{k3}\sigma_{k3}^2\delta_{k3}\beta_{k3} + 2\sigma_{k1}\sigma_{k2}b_{k12}(\delta_{k1}\beta_{k2} + \delta_{k2}\beta_{k1}) \\ &- 2\sigma_{k1}\sigma_{k3}b_{k13}(\delta_{k1}\beta_{k3} + \beta_{k1}\delta_{k3}) + 2\sigma_{k2}\sigma_{k3}b_{k23}(\delta_{k2}\beta_{k3} + \beta_{k2}\delta_{k3}), \end{aligned}$$

$$\begin{aligned} A_{k13} = &\ 2b_{k1}\sigma_{k1}^2\delta_{k1} - 2b_{k2}\sigma_{k2}^2\delta_{k2} + 2b_{k3}\sigma_{k3}^2\delta_{k3} \\ &- 2\sigma_{k1}\sigma_{k2}b_{k12}(\delta_{k1} + \delta_{k2}) + 2\sigma_{k1}\sigma_{k3}b_{k13}(\delta_{k1} + \delta_{k3}) - 2\sigma_{k2}\sigma_{k3}b_{k23}(\delta_{k2} + \delta_{k3}), \end{aligned}$$

$$\begin{aligned} A_{k23} = &-2b_{k1}\sigma_{k1}^2\beta_{k1} - 2b_{k2}\sigma_{k2}^2\beta_{k2} - 2b_{k3}\sigma_{k3}^2\beta_{k3} \\ &+ 2\sigma_{k1}\sigma_{k2}b_{k12}(\beta_{k1} + \beta_{k2}) - 2\sigma_{k1}\sigma_{k3}b_{k13}(\beta_{k1} + \beta_{k3}) + 2\sigma_{k2}\sigma_{k3}b_{k23}(\beta_{k2} + \beta_{k3}), \end{aligned}$$

$$b_{k12} = \frac{1 - e^{-\vartheta(\mu_{k1}+\mu_{k2})}}{\mu_{k1} + \mu_{k2}}, \quad b_{k13} = \frac{1 - e^{-\vartheta(\mu_{k1}+\mu_{k3})}}{\mu_{k1} + \mu_{k2}}, \quad b_{k23} = \frac{1 - e^{-\vartheta(\mu_{k2}+\mu_{k3})}}{\mu_{k2} + \mu_{k3}},$$

$$b_{ki} = \frac{1 - e^{2\vartheta\mu_{ki}}}{2\mu_{ki}}, \quad i = 1, 2, 3.$$

In order to show that the infinite-dimensional vector $V^0(\zeta) = (V_1^0(\zeta), V_2^0(\zeta), \ldots)$ has a finite norm, we write the squared norm as

$$\|V^0\|^2 = \int_{-\vartheta}^{0} \left[\sum_{k=1}^{\infty} \big(V_k^0(\zeta)\big)^2\right] d\zeta = \sum_{k=1}^{\infty} \|V_k^0\|^2$$

and by direct calculation obtain

$$\|V^0\|^2 = \sum_{k=1}^{\infty} \frac{\Delta_{k1}}{\lambda_k^{2\varepsilon} N_k^2 \omega_k^2 [A_{k1}\Delta_{k1} - A_{k12}\Delta_{k2} + A_{k13}\Delta_{k3}]}. \tag{3.10}$$

Therefore, taking into account (2.7) and (3.10), we have

$$|\varphi_k[t, \Delta_k(\tau)]| \le \frac{\delta_k}{\lambda_k^{\varepsilon} N_k \omega_k} \left(\frac{\Delta_{k1}}{A_{k1}\Delta_{k1} - A_{k12}\Delta_{k2} + A_{k13}\Delta_{k3}}\right)^{1/2}. \tag{3.11}$$

Convergence of the series (3.10) can be improved by a suitable choice of $N(x)$ and ε. Thus, in (3.9) we have constructed a function $V_k^0(t, \tau)$ (after the inverse transformation of the variable $\zeta = \tau - t$ in $V_k^0(\zeta)$) which is optimal and universal.

Thus, the optimal operation φ_k^0 that restores $w_k(t)$ with the smallest error is the following:

$$\varphi_k^0[t, y_k(\tau)] = \int\limits_{t-\vartheta}^{t} V_k^0(t,\tau) y_k(\tau)\, d\tau,$$

and the error estimate has the form (3.11).

The resolving operation $\varphi_k[t, \{y_k(\tau), u_k(\tau)\}]$ is determined according to (3.1).

Thus, having the optimal functions $V_k^0(t,\tau)$ in explicit form, as well as the measurement value $y_k(\tau)$, we obtain the most accurate value of $w_k(t)$. Substituting this value of $w_k(t)$ into (2.1), we obtain the desired deflection. Convergence of the corresponding series follows from the convergence of the norm (3.10).

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