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Physics

UNIVERSE EVOLUTION IN THE EINSTEIN FRAME OF JORDAN–BRANS–DICKE THEORY

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Cosmological models of the Universe in the framework of Jordan–Brans–Dicke theory in the Einstein frame in the case of scalar field domination as well as in the presence of cosmological constant Λ with matter, which is described by barometric equation of state $P = \alpha\varepsilon$ (P is the pressure, ε is the energy density of matter), are considered. The analysis of obtained results in the light of observational data are done. It is shown that the contribution of the scalar field and Λ-member ($\Lambda > 0$) in the case of $q = -1/2$ (q is the "decelerating" parameter) compensate each other. In addition, the situation leads to Einstein's theory.

***Keywords*:** relativity and gravitation, dark energy, observational cosmology, cosmological constant.

1. Introduction. In the scales of $\sim 10^8$ light years and more the Universe can be considered as an isotropic and homogeneous structure, for the description of matter of which it is rational to use the model of perfect fluid with the standard energy-momentum tensor. In this work we tried to explain accelerating expansion of the Universe on the comparable simple analytically built model. The problem is solved in the framework of Einstein frame of Jordan–Brans–Dicke (JBD) theory in the presence of cosmological constant Λ.

2. Field Equations. As a result of variation of conformal transformed action of JBD theory [1]

$$W = \frac{1}{c}\int\sqrt{-\overline{g}}\left[-\frac{y_0}{2x}\left(\overline{R} + 2\Lambda\right) + \frac{1}{2}\overline{g}^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta} + L_m\right]d^4x \tag{1}$$

the field equations are presented in the following way:

$$\overline{g}^{\alpha\beta}\overline{\nabla}_\alpha\Phi_{,\beta} = 0, \qquad \overline{G}_{\alpha\beta} - \Lambda\overline{g}_{\alpha\beta} = \frac{x}{y_0}\left(\overline{T}_{\alpha\beta} + \overline{\tau}_{\alpha\beta}\right),$$
$$\overline{\tau}_{\alpha\beta} = \Phi_{,\alpha}\Phi_{\beta} - \frac{1}{2}\overline{g}_{\alpha\beta}\overline{g}^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}, \quad \Phi_{,\alpha} = \frac{y_{,\alpha}}{y}\sqrt{\frac{(3+2\zeta)}{2x}y_0}, \tag{2}$$

where y is the scalar potential of JBD theory.

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It is known, that the geometry of describing model of the Universe is coincided with the metric of Friedmann–Robertson–Wolker (FRW)

$$dS^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta + \sin^2\theta d\varphi^2\right)\right] \tag{3}$$

with $k = -1,0,1$. Thus, eq.-s (2) (in the units of $c = 1$) take a form

$$\frac{d}{dt}\left(\dot{\Phi}a^3\right) = 0, \qquad \dot{\Phi} = c_1\left(\frac{a_0}{a}\right)^3, \tag{4}$$

$$3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = 8\pi G\left(\varepsilon + \frac{1}{2}\dot{\Phi}^2\right) + \Lambda, \tag{5}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi G\left(\alpha\varepsilon + \frac{1}{2}\dot{\Phi}^2\right) + \Lambda, \tag{6}$$

$$\varepsilon(t) = \varepsilon_0\left(\frac{a_0}{a}\right)^n, \quad n = 3(1+\alpha), \tag{7}$$

$$P(t) = \alpha\varepsilon(t), \quad \alpha = -1, 0, \frac{1}{3}, 1, \tag{8}$$

where dot denotes the deviation with respect of time, a_0 is the value scale factor $a(t)$ in the fixed moment of time t_0, and $c_1^2 = \frac{2H_0^2}{8\pi G}(1-q_0) - \varepsilon_0 = \frac{3H_0^2}{8\pi G}\left[\frac{2}{3}(1-q_0) - \overset{0}{\Omega}_m\right]$, which is in GR ($q_o = -\frac{1}{2}$ and $\overset{0}{\Omega}_m = 1$) leads to zero.

Eq.-s (4)–(8) are written in more compact way in the case of known denotations – Habble «constant» $H \equiv \dot{a}/a$ and the relation $\Omega(t) \equiv \varepsilon/\varepsilon_c$, where $\varepsilon_c \equiv 3H^2/8\pi G$ is introduced by analogy of Einstein critical energy density $\overset{0}{\varepsilon}_c = 3H_0^2/8\pi G$.

(8) can be presented in the following way:

$$1 + \frac{k}{a^2H^2} = \frac{8\pi G}{3H^2}\varepsilon + \frac{8\pi G}{3H^2}\cdot\frac{\dot{\Phi}^2}{2} + \frac{\Lambda}{3H^2}, \quad 1 = \frac{\varepsilon}{\varepsilon_c} + \frac{\varepsilon_{ck}}{\varepsilon_c} + \frac{\varepsilon_\Lambda}{\varepsilon_c} - \frac{\varepsilon_k}{\varepsilon_c}, \tag{9}$$

where $\varepsilon_{ck} = \frac{1}{2}c_1^2\left(\frac{a_0}{a}\right)^6$, $\varepsilon_\Lambda = \frac{\Lambda}{8\pi G}$, $\varepsilon_k = \frac{3k}{8\pi Ga^2}$ are respectively energy density of scalar field, field, generating Λ-member and field, generating curvature of space.

Then in such a way, by the introduction of so-called "decelerating" parameter $q = \ddot{a}a/\dot{a}^2$, let us write (9) $2q + 1 = -3\alpha\frac{\varepsilon}{\varepsilon_c} - \frac{3\varepsilon_{ck}}{\varepsilon_c} + \frac{3\varepsilon_\Lambda}{\varepsilon_c} - \frac{\varepsilon_k}{\varepsilon_c}$, therefore, in the summary we get the compacted wrote system equations

$$\Omega_m + \Omega_{ck} + \Omega_\Lambda = 1 + \Omega_k, \quad 2q + 1 = -3\alpha\Omega_m - 3\Omega_{ck} + 3\Omega_\Lambda - \Omega_k. \tag{10}$$

Hear $\Omega_m \equiv \varepsilon/\varepsilon_c$, $\Omega_{ck} \equiv \varepsilon_{ck}/\varepsilon_c$, $\Omega_\Lambda \equiv \varepsilon_\Lambda/\varepsilon_c$, $\Omega_k \equiv \varepsilon_k/\varepsilon_c$. It is worth to write an eq. (10) also in the following way:

$$\frac{2}{3}\cdot\frac{\dot{H}}{H^2}+1=-\alpha\Omega_m-\Omega_{ck}+\Omega_\Lambda-\frac{1}{3}\Omega_k\,. \tag{11}$$

From where it becomes apparent, that when contributions of scalar fields Ω_{ck} and Ω_Λ compensates each other, dynamics of changing of H with time becomes like Einstein. From eq. (10)

$$q=\Omega_\Lambda-2\Omega_{ck}-(1+3\alpha)\frac{\Omega_m}{2} \tag{12}$$

it follows, that in the case of $q=-1/2$, which is until the recent time is considered prospective value of "decelerating" parameter (experiment WMAP [2]), $\Omega_m=1-2\Omega_{ck}$, thus, in the framework of considered model during the certain short period of the lifetime of the Universe, when $\Omega_{ck}=\Omega_\Lambda$ and $q=-1/2$, the decelerating expansion occurs just as in the Einstein theory of Gravity. Then after few years situation is changing as, that q becomes positive [3]. It is naturally enough to assume, that at some the intermediate moment of time a q becomes to zero. According to our estimations meanwhile $\Omega_\Lambda\approx 0{,}52$, $\Omega_{ck}\approx 0{,}18$, if taking as faith $\Omega_m\approx 0{,}3$, estimated in the same works [3] .

3. Model of Matter Domination (α = 0, k = 0). In the work [4] the exact analytical expressions of $a(t)$ are obtained for some cases of state equation. Let's rewrite these relations by taking into account the above mentioned notations.

For $q(t)$ it is convenient to use the formula $q=\frac{3}{2}(\Omega_\Lambda-\Omega_{ck})-\frac{1}{2}$. Scale factor $a(t)$ for the Universe with dust has the following form:

a) $\Lambda>0$. If the condition $4\overset{0}{\Omega}_{ck}\overset{0}{\Omega}_\Lambda>\left(\overset{0}{\Omega}_m\right)^2$ holds, then

$$\left(\frac{a}{a_0}\right)^3=b^+\mathrm{sh}[3H_0\sqrt{\overset{0}{\Omega}_\Lambda}(t-t_0)+\delta^+]-\frac{1}{2}\cdot\frac{\overset{0}{\Omega}_m}{\overset{0}{\Omega}_\Lambda}. \tag{13}$$

In the case of $4\overset{0}{\Omega}_{ck}\overset{0}{\Omega}_\Lambda<\left(\overset{0}{\Omega}_m\right)^2$

$$\left(\frac{a}{a_0}\right)^3=b^-\mathrm{ch}[3H_0\sqrt{\overset{0}{\Omega}_\Lambda}(t-t_0)+\delta^-]-\frac{1}{2}\cdot\frac{\overset{0}{\Omega}_m}{\overset{0}{\Omega}_\Lambda}. \tag{14}$$

For both circumstances by the sign «0» are denoted values, corresponding to the fixed moment of time t_0. The constants have the following forms:

$$(b^+)^2=-(b^-)^2=\frac{\overset{0}{\Omega}_m}{\overset{0}{\Omega}_\Lambda}-\frac{1}{4}\left(\frac{\overset{0}{\Omega}_m}{\overset{0}{\Omega}_\Lambda}\right)^2,\quad e^{\delta^\pm}=\frac{1+\frac{1}{2}\cdot\frac{\overset{0}{\Omega}_m}{\overset{0}{\Omega}_\Lambda}+\sqrt{\left(1+\frac{1}{2}\cdot\frac{\overset{0}{\Omega}_m}{\overset{0}{\Omega}_\Lambda}\right)^2+b^{\pm\,2}}}{b^\pm}. \tag{15}$$

б) $\Lambda<0$. General solution for $a(t)$ followed in this case from the equation [1]

$$\frac{d}{dt}\left(\frac{a}{a_0}\right)^3 = 3H_0\sqrt{\overset{0}{\Omega}_\Lambda}\sqrt{\frac{\overset{0}{\Omega}_{ck}}{|\overset{0}{\Omega}_\Lambda|}+\frac{1}{4}\left(\frac{\overset{0}{\Omega}_m}{\overset{0}{\Omega}_\Lambda}\right)^2-\left[\left(\frac{a}{a_0}\right)^3-\frac{1}{2}\cdot\frac{\overset{0}{\Omega}_m}{|\overset{0}{\Omega}_\Lambda|}\right]^2}\,, \tag{16}$$

where the expression under root should be positive, i.e. only one of possibilities can be realized for the Universe expansion:

$$\left(\frac{a}{a_0}\right)^3 < \frac{1}{2}\overset{0}{\Omega}_m\Big/|\overset{0}{\Omega}_\Lambda| + \sqrt{\overset{0}{\Omega}_{ck}\Big/|\overset{0}{\Omega}_\Lambda| + \frac{1}{4}\left(\overset{0}{\Omega}_m\Big/\overset{0}{\Omega}_\Lambda\right)^2}\,. \tag{17}$$

The solution of eq. (16) has a form

$$\left(\frac{a}{a_0}\right)^3 = \sqrt{\overset{0}{\Omega}_{ck}\Big/|\overset{0}{\Omega}_\Lambda| + \frac{1}{4}\left(\overset{0}{\Omega}_m\Big/\overset{0}{\Omega}_\Lambda\right)^2}\,\sin(3H_0\sqrt{\overset{0}{\Omega}_\Lambda}\,t) + \frac{1}{2}\overset{0}{\Omega}_m\Big/|\overset{0}{\Omega}_\Lambda| \tag{18}$$

and, seemingly, unattractive.

4. Vacuum Model (α = –1, k = 0). From eq. (12) in the case of $\Lambda>0$ the expression for q gets the form $q = 1 - 3\Omega_{ck}$, where in the case of accelerating expansion of the Universe the constraint on the value of scalar field contribution follows

$$q > 0 \Rightarrow \Omega_{ck} < 1/3, \tag{19}$$

mean while from eq. (11) it follows

$$\dot{H} = -3H^2\Omega_{ck}\,. \tag{20}$$

For the scale factor $a(t)$ variation with time we have the following:

$$\left(a(t)/a(t_0)\right)^3 = D_0 \mathrm{sh}[\chi_0(t-t_0)+\delta], \tag{21}$$

$$e^\delta = A_0/D_0\,,\ 1+\sqrt{1+D_0^2} = A_0\,, D_0^2 = \overset{0}{\Omega}_{ck}\Big/\left(\overset{0}{\Omega}_m + \overset{0}{\Omega}_\Lambda\right),\ \chi_0 = 3H_0\sqrt{\overset{0}{\Omega}_m + \overset{0}{\Omega}_\Lambda}\,.$$

In the case of $\Lambda < 0$ equation for description of $\left(a/a_0\right)^3 \equiv x$ has the form

$$\frac{1}{3}\cdot\frac{dx}{dt} = H_0\sqrt{\overset{0}{\Omega}_{ck} + \left(\overset{0}{\Omega}_m - \overset{0}{\Omega}_\Lambda\right)x^2}\,,\quad \overset{0}{\Omega}_\Lambda = \frac{|\Lambda|}{8\pi G\varepsilon_c}\,. \tag{22}$$

Depending on sign of $\left(\overset{0}{\Omega}_m - \overset{0}{\Omega}_\Lambda\right)$, the following solutions are received:

$$\left(\frac{a}{a_0}\right)^3 = \sqrt{\frac{\overset{0}{\Omega}_{ck}}{\overset{0}{\Omega}_\Lambda\overset{0}{\Omega}_m}}\,\sin\left(3H_0\sqrt{\overset{0}{\Omega}_{ck}}\,(t-t_0)\right)+1\,,\quad \overset{0}{\Omega}_\Lambda > \overset{0}{\Omega}_m\,, \tag{23}$$

$$\left(\frac{a}{a_0}\right)^3 = \sqrt{\frac{\overset{0}{\Omega}_{ck}}{\overset{0}{\Omega}_m - \overset{0}{\Omega}_\Lambda}}\,\mathrm{sh}\left(3H_0\sqrt{\overset{0}{\Omega}_{ck}}\,(t-t_0)\right)+1\,,\ \overset{0}{\Omega}_m > \overset{0}{\Omega}_\Lambda\,. \tag{24}$$

5. Model of Stiff Fluid With the State Equation (α = 1, k = 0). From eq. (12) we get $q = 3\Omega_\Lambda - 2$. For accelerating expansion it is true the following

$$q > 0 \Rightarrow \Omega_\Lambda > 2/3\,. \tag{25}$$

From eq. (11) we have

$$\dot{H} = 3H^2(\Omega_\Lambda - 1)\,. \tag{26}$$

For the time dependence on $a(t)$ we get

$$\left(\frac{a}{a_0}\right)^3 = D_6 \mathrm{sh}[\chi_6(t-t_0)+\delta], \tag{27}$$

where $A_6 = 1+\sqrt{1+D_6^2}$, $\dfrac{\overset{0}{\Omega}_{ck}+\overset{0}{\Omega}_m}{\overset{0}{\Omega}_\Lambda} = D_6$, $\chi_6 = 3H_0\sqrt{\overset{0}{\Omega}_\Lambda}$, $e^\delta = A_6/D_6$.

For the light, coming from the stellar object, it is typical redshift, conditioned by the expansion of the Universe. Wavelength λ increases linear proportional to the scale factor $a(t)$. This effect can be taken into account by introducing the concept of red-shift z: $1+z=\lambda_0/\lambda = a_0/a$, where index «0» corresponds to the moment of observation. From this it follows $\dot{z} = -H(1+z)$. This gives a possibility to evaluate the age of the Universe

$$\Delta t_B = \int_0^{t_0} dt = \int_0^\infty \frac{dz}{H(1+z)}. \tag{28}$$

From eq.-s (9) and (19) we get

$$\Delta t_B = \frac{1}{H_0}\int_0^\infty \frac{dz}{(1+z)\sqrt{\overset{0}{\Omega}_{ck}(1+z)^6+\overset{0}{\Omega}_m(1+z)^3-\overset{0}{\Omega}_k(1+z)^2+\overset{0}{\Omega}_\Lambda}}. \tag{29}$$

Replacement of the variable by $y \equiv 1/(1+z)^3$, in the case of conventional conception of flat Universe leads the integral to the following form:

$$\Delta t_B = \frac{1}{3H_0}\int_0^1 \frac{dy}{\sqrt{\overset{0}{\Omega}_\Lambda y^2+\overset{0}{\Omega}_m y+\overset{0}{\Omega}_{ck}}}. \tag{30}$$

In the case of the values of $\Omega_\Lambda = 0$, $\overset{0}{\Omega}_{ck}=0$, $\overset{0}{\Omega}_m=1$ eq. (30) gives the Einstein estimation of the Universe age $\Delta t_B^E = \frac{2}{3}H_0^{-1} \approx 8 \div 10\, Gyr$, where we used the value of H, according to the project of «Habble Space Telescope Key»: $H_o^{-1} = 9{,}77h^{-1}\, Gyr$, $0{,}64 < h < 0{,}8$. It doesn't agree with the limited value of stellar lifetime, which is $>11 \div 12\, Gyr$. Thus, in GR the aging problem exists.

In our case the integral (28) is equal to

$$\Delta t_B = \frac{2}{3H_0 2\sqrt{\overset{0}{\Omega}_\Lambda}} \ln\left(\frac{2\sqrt{\overset{0}{\Omega}_\Lambda}+2\overset{0}{\Omega}_\Lambda+\overset{0}{\Omega}_m}{2\sqrt{\overset{0}{\Omega}_\Lambda\overset{0}{\Omega}_{ck}}+\overset{0}{\Omega}_m}\right) = \frac{\Delta t_B^E}{2\sqrt{\overset{0}{\Omega}_\Lambda}} \ln\left(\frac{2\sqrt{\overset{0}{\Omega}_\Lambda}+2\overset{0}{\Omega}_\Lambda+\overset{0}{\Omega}_m}{2\sqrt{\overset{0}{\Omega}_\Lambda\overset{0}{\Omega}_{ck}}+\overset{0}{\Omega}_m}\right). \tag{31}$$

In the case of values $\Omega_\Lambda = \Omega_{ck}$ the age of the Universe is close to the estimation received in GR:

$$\Delta t_B = \Delta t_B^E \ln\left(1+2\sqrt{\overset{0}{\Omega}_\Lambda}\right) \Big/ 2\sqrt{\overset{0}{\Omega}_\Lambda}. \tag{32}$$

Conclusion. The standard cosmological model is investigated in the framework of “Einstein” frame of JBD theory with the point of view of the contribution of different components’ energy densities. It is shown that for the value $q=-1/2$, estimated in the WMAP experiment [5], contributions of energies, conditioned by the presence of scalar field and Λ-term, compensate each other and the expansion occurs by scheme, like an Einstein. In sequel in the case of violation of this condition q, seems, to be zero in the case of the values $\Omega_m \approx 0{,}3$; $\Omega_\Lambda = 0{,}52$; $\Omega_{ck} = 0{,}18$, and then becomes positive. It is worth to present time dependence of $a(t)$ as a result of this work, and also there are qualitatively described dynamics of the Universe evolution in the limited case of minimal coupled scalar an tensor fields.

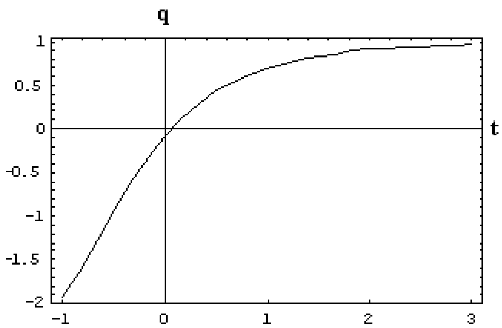

Fig. 1. Theoretical curve of "deceleration" parameter $q(t)$, t is in the units of 4,5 *Gyr* and the zero point corresponds to the moment of observation.

According to our cosmological model the phase transition of the Universe expansion from a decelerating to an accelerating can be realized as shown in Fig. 1.

As it is seen in Fig. 2, the theoretical curve is in agree with the observational data in the case of bigger values of Ω_Λ. Here we used observational data of Ia type Supernovae [6] and showed the dependence of effective stellar magnitude m on z redshift [7].

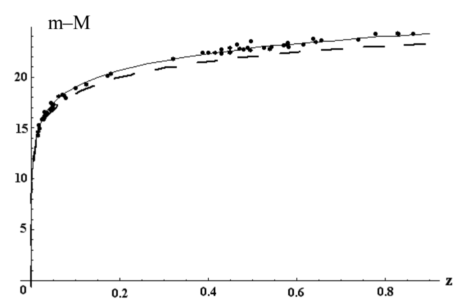

Fig. 2. The black points are the observational data from Ia type Supernovae. Upper continuous curve corresponds to Ω_Λ=0,65, while lower – Ω_Λ=0,5.

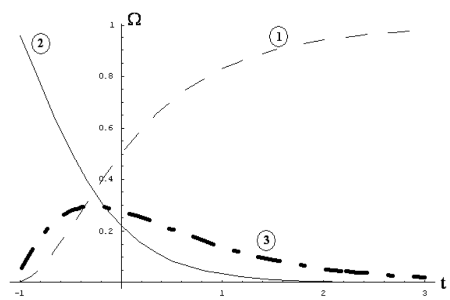

Fig. 3. Zero point corresponds to the moment of observation. Curve (1) represents the dependence Ω_Λ, curve (2) – Ω_{ck} and curve (3) – Ω_m on *t*.

For a more complete picture of the Universe evolution we also present the dependence on time for the different contributions in Fig. 3.

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Տիեզերքի էվոլյուցիան Յորդան–Բրանս–Դիկեի
տեսության էյնշտեյնյան ներկայացմամբ

Հոդվածում դիտարկվում է Տիեզերքի մոդելը Յորդան–Բրանս–Դիկեի տեսության էյնշտեյնյան ներկայացման շրջանակներում, երբ առկա են սկալյար դաշտը և Λ-կոսմոլոգիական հաստատունը, իսկ նյութի վիճակը նկարագրվում է բարոմետրական հավասարումով՝ $P=\alpha\varepsilon$ (P-ն ճնշումն է, ε-ը՝ նյութի էներգիայի խտությունը)։ Ստացված վերլուծական արդյունքները համեմատված են արդի դիտարկումների տվյալների հետ։ Ցույց է տրված, որ սկալյար դաշտի և Λ ($\Lambda>0$) անդամի ներդրումները $q=-1/2$ դեպքում (q-ն “դանդաղեցման” պարամետրն է) իրար համակշռում են և արդյունքը նմանվում է էյնշտեյնյան դեպքին։

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Эволюция Вселенной в эйнштейновском представлении теории Йордана–Бранса–Дикке

Космологическая модель Вселенной рассматривается в рамках эйнштейновского представления теории Йордана–Бранса–Дикке в условиях доминирования скалярного поля, а также при наличии космологической постоянной Λ с веществом, описываемым барометрическим уравнением состояния $P=\alpha\varepsilon$ (P – давление, ε – плотность энергии материи). Выполнен анализ полученных аналитических результатов в свете современных данных наблюдений. Показано, что вклады скалярного поля и Λ-члена ($\Lambda>0$) при $q=-1/2$ (q – параметр “замедления”) компенсируют друг друга и ситуация оказывается подобной эйнштейновской.