ЕРЕВАНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
YEREVAN STATE UNIVERSITY

СТУДЕНЧЕСКОЕ НАУЧНОЕ ОБЩЕСТВО
STUDENT SCIENTIFIC SOCIETY

ISSN 1829-4367

СБОРНИК НАУЧНЫХ СТАТЕЙ СНО ЕГУ

COLLECTION OF SCIENTIFIC ARTICLES OF YSU SSS

1.1 (24)

Естественные и физико-математические науки
(География и геология, информатика и прикладная математика, биология, математика и механика, химия, фармацевтика, физика)

Natural and Physical-Mathematical Sciences
(Geography and Geology, Informatics and Applied Mathematics, Biology, Mathematics and Mechanics, Chemistry, Pharmacy, Physics)

ЕРЕВАН - YEREVAN
ИЗДАТЕЛЬСТВО ЕГУ - YSU PRESS
2018

ԵՐԵՎԱՆԻ ՊԵՏԱԿԱՆ ՀԱՄԱԼՍԱՐԱՆ
ՈՒՍԱՆՈՂԱԿԱՆ ԳԻՏԱԿԱՆ
ԸՆԿԵՐՈՒԹՅՈՒՆ

ISSN 1829-4367

ԵՊՀ ՈՒԳԸ ԳԻՏԱԿԱՆ ՀՈԴՎԱԾՆԵՐԻ ԺՈՂՈՎԱԾՈՒ

1.1 (24)

Բնական և ֆիզիկամաթեմատիկական գիտություններ
(աշխարհագրություն և երկրաբանություն, ինֆորմատիկա և կիրառական մաթեմատիկա, կենսաբանություն, մաթեմատիկա և մեխանիկա, քիմիա, ֆարմացիա, ֆիզիկա)

ԵՐԵՎԱՆ
ԵՊՀ ՀՐԱՏԱՐԱԿՉՈՒԹՅՈՒՆ
2018

Հրատարակվում է ԵՊՀ գիտական խորհրդի որոշմամբ
Издается по решению Ученого совета ЕГУ
Published by the resolution of the Academic Council of YSU

Հրատարակիչ՝ ԵՊՀ հրատարակչություն
Հասցե՝ ՀՀ, ք. Երևան, Ալ. Մանուկյան 1, (+374 10) 55 55 70, publishing@ysu.am

Հրատարակության նախապատրաստող ստորաբաժանում՝ ԵՊՀ ուսանողական գիտական ընկերություն
Հասցե՝ ՀՀ, ք. Երևան, Ալ. Մանուկյան 1, (+374 60) 71 01 94,
Էլ. փոստ՝ sss@ysu.am
ԵՊՀ ՈՒԳԸ հրատարակումների կայք՝ www.ssspub.ysu.am.

Kalichava Zviad
Iv. Javakhishvili Tbilisi State University
Department of Computer Science, PhD Student
E-mail: zviadi.kalichava@gmail.com

THE EXACTNESS OF AN ALGORITHM STEP FOR A DYNAMIC BEAM

AMS subject classifications: 65M60, 65M15.

1. Problem statement

Let us consider the nonlinear differential equation

$$\frac{\partial^2 u}{\partial t^2}(x,t)+\frac{\partial^4 u}{\partial x^4}(x,t)-h\frac{\partial^4 u}{\partial x^2\,\partial t^2}(x,t)-\left(\lambda+\int_0^L\left(\frac{\partial u}{\partial \xi}(\xi,t)\right)^2 d\xi\right)\frac{\partial^2 u}{\partial x^2}=f(x,t), \qquad (1.1)$$

$$0<x<L, \qquad 0<t\le T,$$

with the initial boundary conditions

$$u(x,0)=u^0(x), \qquad \frac{\partial u}{\partial t}(x,0)=u^1(x),$$

$$u(0,t)=u(L,t)=0, \qquad \frac{\partial^2 u}{\partial x^2}(0,t)=\frac{\partial^2 u}{\partial x^2}(L,t)=0, \qquad (1.2)$$

where $h\geq 0$ and $\lambda>0$ are some given constants, $u^0(x)$, $u^1(x)$ and $f(x,t)$ are the given functions and $u(x,t)$ is the function we want to obtain.

Equation (1.1) is given in E. Henriques de Brito [2] and belongs to the class of equations based on the Timoshenko theory [6]. Here we consider one of the questions of approximate algorithms for equation (1.1). Numerical algorithms for nonlinear integro-differential dynamic beam equations are investigated in [1, 3, 4, 5, 7].

2. Solvability

In [2], the existence and uniqueness of a generalied solution of the Cauchy problem is proved for the equation

$$(I+hA)u''+A^2u+\left[\lambda+M\left(\left|A^{\frac{1}{2}}u\right|^2\right)\right]Au=f$$

a particular case of which is equation (1.1). Let us apply the result of [2] to our case.

The symbol (,) will be understood as a scalar product in $L_2(0,L)$. Let $\overset{0}{W}{}_2^2(0,L)$ consist of functions of the spase $\overset{0}{W}{}_2^2(0,L)\cap W_2^2(0,L)$.

According to [2], if

$$u^0(x)\in \overset{0}{W}{}_2^2(0,L), \quad u^1(x)\in \overset{0}{W}{}_2^1(0,L), f(x,t)\in L_2\left(0,L;\ \overset{0}{W}{}_2^1(0,L)\right), \qquad (2.1)$$

then there is a unique function $u = u(x,t)$,

$$u \in L^{\infty}\left(0,T;\ \overset{0}{W_2^2}(0,L)\right), \qquad \frac{\partial u}{\partial t} \in L^{\infty}\left(0,T;\ W_2^1(0,L)\right), \tag{2.2}$$

such that u is a weak solution of problem (1.1), (1.2), i.e. for every $v = v(x) \in \overset{0}{W_2^2}(0,L)$,

$$\frac{d}{dt}\left[\left(\frac{\partial u}{\partial t}, v\right) + h\left(\frac{\partial^2 u}{\partial x\,\partial t}, \frac{\partial v}{\partial x}\right)\right] + \left(\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 v}{\partial x^2}\right) + \left(\lambda + \int_0^L \left(\frac{\partial u}{\partial x}\right)^2 dx\right)\left(\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}\right) = (f, v) \tag{2.3}$$

and

$$u(x,0) = u^0(x), \quad \frac{\partial u}{\partial t}(x,0) = u^1(x). \tag{2.4}$$

Let us make the conclusions needed in the sequel.

With (2.2) taking into account, we can write the solution of the problem (1.1), (1.2) in the form

$$u(x,t) = \sum_{i=1}^{\infty} u_i(t) \sin\frac{i\pi x}{L}. \tag{2.5}$$

Let us also represent $u^0(x), u^1(x)$, $f(x,t)$ functions by the series

$$u^l(x) = \sum_{i=1}^{\infty} u_i^{(l)} \sin\frac{i\pi x}{L}, \quad l = 0,1, \qquad f(x,t) = \sum_{i=1}^{\infty} f_i(t) \sin\frac{i\pi x}{L}. \tag{2.6}$$

As follows from (2.1)-(2.4), after replacing the function v by the function $\sin\frac{i\pi x}{L}$, $i = 1, 2, \dots$, the coefficients $u_i(t)$ in (2.5) satisfy the system of ordinary differential equations $\left(1 + h\left(\frac{\pi i}{L}\right)^2\right) u_i''(t) + \left(\frac{\pi i}{L}\right)^4 u_i(t) + \left(\lambda + \frac{L}{2}\sum_{j=1}^{\infty}\left(\frac{\pi j}{L}\right)^2 u_j^2(t)\right)\left(\frac{\pi i}{L}\right)^2 u_i(t) = f_i(t)$,

$$i = 1, 2, \dots, \quad 0 < t \le T,$$

with the initial conditions

$$u_i(0) = u_i^{(0)}, \quad u_i'(0) = u_i^{(1)}, \qquad i = 1, 2, \dots,$$

where in (2.6)

$$f_i(t) = \frac{2}{L}\int_0^L f(x,t) \sin\frac{i\pi x}{L}\, dx,$$

$$u_i^{(0)} = \frac{2}{L}\int_0^L u^0(x)\sin\frac{i\pi x}{L}\,dx, \qquad u_i^{(1)} = \frac{2}{L}\int_0^L u^1(x)\sin\frac{i\pi x}{L}\,dx\,.$$

Finally, in view of (2.5) and (2.2) we conclude that the series $\sum_{i=1}^{\infty} i^4 u_i^2(t)$ and $\sum_{i=1}^{\infty} i^2 u_i'^2(t)$ converge.

3. Algorithm

a. Galerkin method. An approximate solution of problem (1.1), (1.2) is written in the form

$$u_n(x,t) = \sum_{i=1}^{n} u_{ni}(t)\sin\frac{i\pi x}{L},$$

where the coefficients $u_{ni}(t)$ are defined by Galerkin method from the system of nonlinear differential equations

$$\left(1 + h\left(\frac{\pi i}{L}\right)^2\right)u_{ni}''(t) + \left(\frac{\pi i}{L}\right)^4 u_{ni}(t) + \left(\lambda + \frac{L}{2}\sum_{j=1}^{\infty}\left(\frac{\pi j}{L}\right)^2 u_{nj}^2(t)\right)\left(\frac{\pi i}{L}\right)^2 u_{ni}(t) = f_i(t), \qquad (3.1)$$

$$i = 1,2,\dots,n, \qquad 0 < t \le T,$$

and the conditions

$$u_{ni}(0) = u_i^{(0)}, \quad u_{ni}'(0) = u_i^{(1)}, \quad i = 1,2,\dots,n. \qquad (3.2)$$

b. Difference scheme. Problem (3.1), (3.2) will be solved using the difference method. On the time interval $[0,T]$ we introduce a net with step $\tau = \frac{T}{M}$ and nodes $t_m = m\tau$, $m = 0,1,\dots,M$. In the m-th node, i.e. for $t = t_m$, the approximate values of $u_{ni}(t)$ is denoted by u_{ni}^m. We use a symmetric difference scheme $\left(1 + h\left(\frac{i\pi}{L}\right)^2\right)\left(\frac{u_{ni}^{m+1} - 2u_{ni}^m + u_{ni}^{m-1}}{\tau^2}\right) +$

$$\left\{\left(\frac{i\pi}{L}\right)^4 + \left(\frac{i\pi}{L}\right)^2\left[\lambda + \frac{L}{4}\sum_{j=1}^{n}\left(\frac{j\pi}{L}\right)^2\left(\left(\frac{u_{nj}^{m+1} + u_{nj}^m}{2}\right)^2 + \left(\frac{u_{nj}^m + u_{nj}^{m-1}}{2}\right)^2\right)\right]\right\}\frac{u_{ni}^{m+1} - 2u_{ni}^m + u_{ni}^{m-1}}{4} = f_i^m, \qquad (3.3)$$

$$m = 1,2,\dots,M-1, \qquad i = 1,2,\dots,n,$$

with conditions

$$u_{ni}^0 = u_i^{(0)}, \;\; u_{ni}^1 = u_i^{(0)} + \tau u_i^{(1)}, \;\; i = 1,2,\dots,n, \qquad (3.4)$$

In (3.3) $f_i^m = f_i(t_m), \quad m = 1,2,\dots,M, \quad i = 1,2,\dots,n,$

c. Iteration method. Let us introduce the notation

$$v_{ni}^{m+1} = \frac{u_{ni}^{m+1} - u_{ni}^m}{\tau}, \qquad w_{ni}^{m+1} = \frac{i\pi}{L}\frac{u_{ni}^{m+1} + u_{ni}^m}{2}$$

and rewrite system (3.3), (3.4) as follows

$$\left(1+h\left(\frac{i\pi}{L}\right)^2\right)\left(\frac{v_{ni}^{m+1}-v_{ni}^{m}}{\tau^2}\right)+$$

$$\left\{\left(\frac{i\pi}{L}\right)^3+\frac{i\pi}{L}\left[\lambda+\frac{L}{4}\sum_{j=1}^{n}\left(\frac{j\pi}{L}\right)^2\left(\left(w_{nj}^{m+1}\right)^2+\left(w_{nj}^{m}\right)^2\right)\right]\right\}\frac{w_{ni}^{m+1}+w_{ni}^{m}}{2}=f_i^m\,, \qquad (3.5)$$

$$\frac{w_{ni}^{m+1}-w_{ni}^{m}}{\tau}=\frac{i\pi}{L}\frac{v_{ni}^{m+1}+v_{ni}^{m}}{2}\,, \qquad m=1,2,\dots,M-1. \qquad (3.6)$$

$$v_{ni}^{1}=u_i^{(1)},\ w_{ni}^{1}=\frac{i\pi}{L}\left(u_i^0+\frac{1}{2}\tau u_i^1\right).$$

On expresing v_{ni}^{m+1} in (3.6) through v_{ni}^{m}, w_{ni}^{m} and $w_{ni}^{m+1}v_{ni}^{m+1}=-v_{ni}^{m}+\frac{2L}{i\pi}\frac{w_{ni}^{m+1}+w_{ni}^{m}}{\tau}$

and substituting into (3.5), we come to the system of equations

$$\varphi_i\left(w_{n1}^{m},w_{n2}^{m},\dots,w_{nn}^{m}\right)=0, \qquad (3.7)$$

$$m=1,2,\dots,M-1,\ \ i=1,2,\dots,n,$$

where

$$\varphi_i=\left(1+h\left(\frac{i\pi}{L}\right)^2\right)\frac{2L}{\tau^2 i\pi}w_{ni}^{m}+$$

$$\left\{\left(\frac{i\pi}{L}\right)^3+\frac{i\pi}{L}\left[\lambda+\frac{L}{4}\sum_{j=1}^{n}\left(\frac{j\pi}{L}\right)^2\left(\left(w_{nj}^{m}\right)^2+\left(w_{nj}^{m-1}\right)^2\right)\right]\right\}\frac{w_{ni}^{m}}{2}+\frac{i\pi}{L}\frac{L}{4}\left(\sum_{j=1}^{n}\left(w_{nj}^{m}\right)^2\right)\frac{w_{ni}^{m-1}}{2}+\psi_i$$

and

$$\psi_i=-\left(1+h\left(\frac{i\pi}{L}\right)^2\right)\frac{v_{nj}^{m-1}}{\tau}+\left\{\left(\frac{i\pi}{L}\right)^3+\frac{i\pi}{L}\left[\lambda+\frac{L}{4}\sum_{j=1,j\neq i}^{n}\left(\left(w_{nj}^{m-1}\right)^2\right)\right]\right\}\frac{w_{ni}^{m-1}}{2}-f_i^m.$$

System (3.7) will be solved node-by-node. Assuming that the solution has already been obtained at (m-1)-th node, to find it at the m-th node we use the Newton iteration method

$$w_{nk+1}^{m}=w_{nk}^{m}-J^{-1}(w_{nk}^{m})\,\varphi(w_{nk}^{m}).$$

where $w_{nk}^{m}=(w_{nik}^{m})_{i=1}^{n}$ is the k-th iteration approximation of $w_{n}^{m}=(w_{ni}^{m})_{i=1}^{n}$ and $J(w_{nk}^{m})$ is the Jacobi matrix.

4. Galerkin method error

Assume that in (2.6)

$$u_i^{(0)^2} \le \frac{\omega_0}{i^{p_0+5}}, \quad u_i^{(1)^2} \le \frac{\omega_1}{i^{p_1+3}}, \qquad i = 1,2,\dots, \tag{4.1}$$

and

$$f_i^2(t) \le \frac{\omega}{i^{p+1}}, \qquad 0 \le t \le T,\ i = 1,2,\dots \tag{4.2}$$

Here $p_l,\ \omega_l,\ = 0,1,\ and\ p, \omega\ are\ some\ positive\ numbers.$

Now our aim is to estimate the error of the Galerkin method, which the defined as follows

$$\delta_n(x,t) = u_n(x,t) - u(x,t).$$

By the coefficients of decomposition (2.5), we form the function

$$\pi_n u(x,t) = \sum_{i=1}^{n} u_i(t) \sin\frac{i\pi x}{L}.$$

Below, we will denote by || || the norm in the space $L_2(0,L)$.

Let us formulate the main result.

Theorem. The inequality

$$\left\|\left(u_n(x,t)-u(x,t)\right)_t\right\|^2 + h\left\|\left(u_n(x,t)-u(x,t)\right)_{xt}\right\|^2 + \left\|\left(u_n(x,t)-u(x,t)\right)_{xx}\right\|^2 + \\ +\lambda\left\|\left(u_n(x,t)-u(x,t)\right)_x\right\|^2 \le r_n(t) + \frac{1}{2}\left(\int_0^T \sigma_1(t) r_n^2(t)dt\right) exp\left(\int_0^t \sigma_2(\tau)d\tau\right)$$

(4.3)

is fulfilled for the error of the Galerkin method.

Here

$$r_n = 2\left(\psi_n + \frac{\alpha_2^2}{\alpha_0\alpha_1^2}\int_0^t \frac{1}{c_1^2(\tau)}\|\Delta_n f(x,\tau)\|^2 d\tau\right) exp\left(\alpha_0\alpha_1^2\int_0^t c_1^2(\tau)d\tau\right),$$

$$\psi_n = \|\Delta_n u^1(x)\|^2 + \|\Delta_n u^{0''}(x)\|^2 + h\|\Delta_n u^{1'}(x)\|^2 + (\lambda + \|u^{0'}(x)\|^2)\|\Delta_n u^{0'}(x)\|^2,$$

$\sigma_1(t) = \frac{1}{2\alpha_0}\alpha_1^2\alpha_2^2\frac{c_1^2(t)}{c_1^2(t)+c_3^2(t)}$, $\sigma_2(t) = \alpha_1\left(\alpha_0\alpha_1\left(c_1^2(t)+c_3^2(t)\right) + \alpha_2(c_0(t)+c_2(t))(c_1(t)+c_3(t))\right)$,

$$\Delta_n u^l(x) = \sum_{i=n+1}^{\infty} u_i^{(l)} \sin\frac{i\pi x}{L}, \quad l = 0,1, \qquad \Delta_n f(x,t) = \sum_{i=n+1}^{\infty} f_i(t) \sin\frac{i\pi x}{L},$$

$\alpha_0 = \frac{1}{1+(h\lambda)^{\frac{1}{2}}}$, $\alpha_1 = \left(\frac{1}{1+\lambda\left(\frac{L}{\pi}\right)^2}\right)^{\frac{1}{2}}\frac{L}{\pi}$, $\alpha_2 = \left(\frac{1}{1+h\left(\frac{\pi}{L}\right)^2}\right)^{\frac{1}{2}}$, $c_0(t) = \left(\left(\left(\frac{\pi}{L}\right)^4 + 2\lambda\left(\frac{\pi}{L}\right)^2 + 2ke^t\right)^{\frac{1}{2}} - \left(\frac{\pi}{L}\right)^2 - \lambda\right)^{\frac{1}{2}}$,

$$c_1(t)=(ke^t)^{\frac{1}{2}},\ \ c_2(t)=\left(\left(\left(\frac{\pi}{L}\right)^4+2\lambda\left(\frac{\pi}{L}\right)^2+2k_ne^t\right)^{\frac{1}{2}}-\left(\frac{\pi}{L}\right)^2-\lambda\right)^{\frac{1}{2}},\ \ c_3(t)=(k_ne^t)^{\frac{1}{2}},$$

$$k=\|u^1(x)\|^2+\left\|u^{0''}(x)\right\|^2+h\left\|\left\|u^{1'}(x)\right\|^2\right\|+\frac{1}{2}\left(\lambda+\left\|u^{0'}(x)\right\|^2\right)^2+\alpha_2^2\int_0^T\|f(x,t)\|^2\,dt,$$

$$k_n=\|\pi_nu^1(x)\|^2+\|\pi_nu^{0''}(x)\|^2+h\|\|\pi_nu^{1'}(x)\|^2\|+\frac{1}{2}(\lambda+\|\pi_nu^{0'}(x)\|^2)^2$$
$$+\alpha_2^2\int_0^T\|\pi_nf(x,t)\|^2\,dt,$$
$$\pi_nu^l(x)=\sum_{i=1}^n u_i^{(l)}\,sin\frac{i\pi x}{L},\ \ l=0,1,\quad \pi_nf(x,t)=\sum_{i=1}^n f_i(t)\,sin\frac{i\pi x}{L}.$$

Using (4.1), (4.2) it is possible to derive formulas, which allow to estimate right side of inequality (4.3) by parameters $p_l,\omega_l,\ \ l=0,1,\ p,\ \omega\ and\ n.$ Same formulas can be also obtained in case, when $u_i^{(l)},\ l=0{,}1$, and $f_i(t)$ change by a rule different from (4.1), (4.2).

BIBLIOGRAPHY

[1] **Bernardi C., Copetti M.**, Discretization of a Dynamic Termoviscoelastic Timoshenko Beam, ZAMM, Wiley-VCH Verlag, 97 (5), 2015, pp. 532-549.

[2] **Henriques de Brito E.**, A Nonlinear Hyperbolic Equation. International Journal of Mathematics and Mathematical Sciences, 3, 1980, pp. 505-520.

[3] **Kalichava Z., Peradze J.**, Approximation with Respect to the Spatial Variable of the Solution of a Nonlinear Dynamic Beam Problem, SCCTW' 2016, South-Caucasus Computing and Technology Workshop, 2016, 15 pages.

[4] **Peradze J.**, On the Accuracy of the Galerkin Method for a Nonlinear Dynamic Beam Equation, Math. Meth. Appl. Sci., vol. 34, issue 14, 2011, pp. 1725-1732.

[5] **Peradze J.**, The Existence of a Solution and a Numerical Method for the Timoshenko Nonlinear Wave System, M2AN, Mathematical Modelling and Numerical Analysis, 38 (1), 2004, pp. 1-26.

[6] **Timoshenko S. P., Young D. H., Weaver W.**, Vibration Problems in Engineering, John Wiley & Sons, Inc., New York, 1974.

[7] **Tucsnak M.**, On an Initial and Boundary Value Problem for the Nonlinear Timoshenko Beam. Anais da Academia Brasileira de Gendas, 63 (2), 1991, pp. 115-125.

Kalichava Zviad

THE EXACTNESS OF AN ALGORITHM STEP FOR A DYNAMIC BEAM

Key words: beam equation, numerical algorithm, Galerkin method, error estimate.

The initial boundary value problem is posed for the Timoshenko type nonlinear integro-differential inhomogeneous equation, which describes the dynamic behaviour of a beam. A numerical algorithm is proposed for the solution of the problem. One of the parts of the algorithm is the Galerkin method, the error of which is estimated.

Կալիչավա Զվիադ

ԱԼԳՈՐԻԹԻ ՔԱՅԼԻ ՃՇԳՐՏՈՒԹՅՈՒՆԸ ԴԻՆԱՄԻԿ ՃԱՌԱԳԱՅԹԻ ՀԱՄԱՐ

Բանալի բառեր՝ ճառագայթային հավասարում, թվային ալգորիթմ, Գալերկինի մեթոդ, սխալականության գնահատում:

Նախնական սահմանանշանակիչ առաջադրանք է տրվել Տիմոշենկոյի տիպի ոչ գծային, ինտեգրո-դիֆերենցիալ հավասարմանը, որը նկարագրում է դինամիկ ճառագայթը: Խնդրի լուծման համար առաջարկվում է թվային ալգորիթմ: Ալգորիթմի մասերց մեկն է Գալերկինի մեթոդը, որի սխալականությունը գնահատվում է մեր հոդ-վածում:

Каличава Звиад

ТОЧНОСТЬ АЛГОРИТМИЧЕСКОГО ШАГА ДЛЯ ДИНАМИЧЕСКОГО ЛУЧА

Ключевые слова: лучевое уравнение, численный алгоритм, метод Галеркина, оценка погрешности.

Начальная гранично-определенная задача задана нелинейному неоднородному интегро-дифференциальному уравнению типа Тимошенко, которое описывает динамическое поведение луча. Для решения задачи предложен численный алгоритм. Одним из частей алгоритма является метод Галеркина, ошибочность которого измерена в нашей статье.