YSU Department of Mathematics and Mechanics

Applied Statistics and Data Science

Entrance Exam Test, Round 1

Exam Date/Time: 07 July 2025, 10:00 - 12:00

Last Name: ______________________ **First Name:** ______________________

Email Address: ______________________________

READ THESE INSTRUCTIONS CAREFULLY

- This test consists of 4 Show-Work Problems and 10 Multiple Choice Problems. Multiple Choice problems can have several correct answers. Please mark the correct answers clearly, avoiding erasing and/or changing your answers several times. Note that unclear responses won't be counted.
- Each Show-Work Problem will be graded 10 points and each Multiple Choice Problem will be graded 6 points.
- This is a closed-book test, and no notes, assignments, practice problems, books, formula sheets or other materials are allowed.
- The use of mobile phones or any other electronic devices are strongly prohibited. Only ordinary calculators are allowed. Please turn off your cell phones and place them out of reach.
- Talking to another student, looking at another student's paper, or communicating with other students in any way is strictly forbidden.
- Use the scratch pages of the test booklet to do your draft calculations. Please ask proctors for extra scratch papers if necessary.
- If you run out of the space on the test pages, please use a scratch page to finish your work. Indicate in the test page that you will continue on the scratch page, and mark with the rectangle the portion on the scratch page that contains the solution. Any other work on the scratch page will not be graded.
- Good luck!

MCh	ShW1	ShW2	ShW3	ShW4	Total

DO NOT OPEN THIS BOOKLET
UNTIL YOU HAVE BEEN TOLD TO DO SO

Scratch Paper

Part 1 - Multiple Choice Problems

1. The function $f(x) = x^2 \ln x$ is

   (A) concave up on $(0, e^{-3/2})$
   (B) concave down on $(-\infty, e^{-3/2})$
   (C) concave up on $(e^{-3/2}, +\infty)$
   (D) increasing on $(0, e^{-1/2})$
   (E) increasing on $(-\infty, 0)$ and $(e^{-1/2}, \infty)$

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2. The limit

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{x \sin x} \right)$$

   is equal to

   (A) 0 (B) ∞ (C) 1 (D) 3 (E) $-\dfrac{1}{6}$

3. Find the tangent plane to the surface $z = 2\sqrt[3]{1+6x} + 6e^{3y-9} + 2$ at the point (0,3,10).

   (A) $z = 4x + 18y - 44$
   (B) $z = 12x + 18y - 44$
   (C) $z = \frac{2}{3}x + 3y + 1$
   (D) $z = 12x + 6y - 8$
   (E) $z = 4x + 6y - 8$

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4. Perform the change of variables $u = x, v = \dfrac{y}{x}$ in the double integral $\iint_D f(x,y)dxdy$, where D is the triangle with vertices $(0,0),(1,0),(1,1)$ and express in terms of new variables.

   (A) $\iint_{[0,1]^2} f(u,uv) \cdot u\, du\, dv.$
   (B) $\iint_D f(u,uv)\, du\, dv.$
   (C) $\iint_D f(u,uv) \cdot u\, du\, dv.$
   (D) $\iint_D f(u,v) \cdot u\, du\, dv.$
   (E) $\iint_{[0,1]^2} f(u,uv)\, du\, dv.$

5. The Taylor series of the function $f(x) = \ln \frac{2+x}{2-x}$ at $x_0 = 0$ is

(A) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$

(B) $-\sum_{n=1}^{\infty} \frac{1}{n} x^n$

(C) $2 \sum_{k=0}^{\infty} \frac{x^{2k+1}}{2k+1}$

(D) $\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1) \cdot 2^{2k}}$

(E) $2 \sum_{k=1}^{\infty} \frac{x^{2k}}{2k}$

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6. Let A be a 2×2 matrix given by

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Find $(I + A)^{100}$, where I is the 2×2 identity matrix.

(A) $I + A$
(B) A
(C) I
(D) $-2^{50} I$
(E) $2^{50}(I + A)$

7. Find the rank of the 2025×2025 matrix $A = (a_{ij})_{i,j=1}^{2025}$, where $a_{ij} = j + 2025(i-1)$.

(i.e. $A = \begin{bmatrix} 1 & 2 & \dots & 2025 \\ 2026 & 2027 & \dots & 4050 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & 2025^2 \end{bmatrix}$)

(A) 1 (B) 2 (C) 3 (D) 2024 (E) 2025

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8. Assume A and B are some events in some probability space, and we know only that

$$\mathbb{P}(A) = 0.5 \qquad \text{and} \qquad \mathbb{P}(B) = 0.7.$$

Please mark below all statements, which are correct for any such A and B.

(A) $\mathbb{P}(A \cup B) = 1$

(B) $\mathbb{P}(A \cap B) > 0$

(C) $\mathbb{P}(B \setminus A) \geq 0.2$, where $B \setminus A$ is the set of all elements of B which are not in A

(D) $\mathbb{P}(A \setminus B) = 0$

(E) $\mathbb{P}(A^c) = \mathbb{P}(A)$, where A^c is the complement of A.

9. Assume A and B are some events in some probability space, and we know only that

$$\mathbb{P}(A) \neq 0 \qquad \text{and} \qquad \mathbb{P}(B) \neq 0.$$

Please mark below all statements, which are correct for any such A and B.

(A) $\mathbb{P}(A|B) > \mathbb{P}(A)$

(B) $\mathbb{P}(A|B) = \mathbb{P}(B|A)$

(C) $\mathbb{P}(A|B) \cdot \mathbb{P}(B) = \mathbb{P}(B|A) \cdot \mathbb{P}(A)$

(D) $\mathbb{P}(A|A) = 0$

(E) $\mathbb{P}(A|A) = \mathbb{P}(A)$

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10. Assume $X_1, X_2, ..., X_n \sim Unif[0,1]$ are independent. Please mark below all correct statements:

(A) $\dfrac{X_1 + X_2 + ... + X_n}{n} = \mathbb{E}(X_1)$

(B) $\mathbb{E}\left(\dfrac{X_1 + X_2 + ... + X_n}{n}\right) = \mathbb{E}(X_1)$

(C) $Var\left(\dfrac{X_1 + X_2 + ... + X_n}{n}\right) = Var(X_1)$

(D) $\dfrac{X_1 + X_2 + ... + X_n}{n} \sim \mathcal{N}(0,1)$

(E) $\sqrt{n} \cdot \left(\dfrac{X_1 + X_2 + ... + X_n}{n} - \dfrac{1}{2}\right)$ is asymptotically (approximately) $\mathcal{N}(0,1)$ distributed.

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Part 2 - Show-Work Problems

1. Let X and Y be independent $Exp(3)$ random variables.

   a. Find, with a proof, the distribution of $3X$ (you can describe its PDF or CDF)

   b. Calculate $\mathbb{E}\left(X^2\right)$

   c. Write down the Joint PDF of X and Y

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2. Let

$$A = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 5 \end{bmatrix},$$

a. Find the eigenvalues of A.

b. What are the eigenvalues of $A + 3I$, where I is the identity matrix of size 3×3?

c. Find eigenvectors v_1, v_2, v_3 of the matrix A, which form a basis for $\mathbb{R}^3$.

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3. Let $f(x, y) = x^3 + 5x + y^2 - 4xy$
    - Find the stationary points of f
    - Check whether those points are extrema points.

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4. Prove that

$$\mathbb{E}\left(\frac{XY}{X^2+Y^2}\right) \geq 0$$

for independent and identically distributed X and Y.

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